H2 Physics
Rotational Motion

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Brain Dump

What do you know about Rotational Motion?

Brain Dump (CONT'D)

What do you know about Rotational Motion?

- Torque $\tau = F \times r_\perp$ (moment of a force)
- Couple: two equal, opposite, parallel forces producing pure rotation
- Centre of gravity and moment of a force
- Equilibrium conditions: $\sum F = 0$, $\sum \tau = 0$
- Moment of inertia I (rotational inertia)
- Angular acceleration $\alpha = \tau / I$
- Rotational kinetic energy $K_{\text{rot}} = \frac{1}{2} I \omega^2$
- Angular momentum $L = I\omega$
- Conservation of angular momentum
- Rolling motion (combination of translation and rotation)

Math Checklist

Before tackling Rotational Motion, ensure you are comfortable with:

- Trigonometry (sin, cos, tan)
- Vector cross products (direction of torque)
- Solving simultaneous equations
- Quadratic equations
- Differentiation and integration (for variable ω)
- Radian measure
- Parallel axis theorem (if needed)
- Integration for moment of inertia (basic)
- Conservation laws
- Energy conservation with rotational terms

Building Intuition – Real-world Applications

- Seesaw: balancing requires equal moments on both sides.
- Wrench: longer handle gives more torque for the same force.
- Opening a door: force applied farthest from hinges is most effective.
- Flywheel: stores rotational energy to smooth engine pulses.
- Spinning figure skater: pulls arms in to spin faster (conservation of angular momentum).
- Bicycle wheel: gyroscopic effect helps with stability.
- Rolling ball: combination of translation and rotation; energy split between K_t and K_r.

Formalization – Torque and Moment of a Force

Torque (Moment of a Force)

$$\tau = F \times r_{\perp} = Fr \sin\theta$$

where r is the distance from pivot to point of force application, θ is angle between force and lever arm.

Couple

Two equal and opposite forces, not collinear, produce a torque $\tau = F \times d$ (where d is perpendicular distance between lines of action). A couple produces pure rotation with no net force.

Centre of Gravity

The point where the entire weight of an object can be considered to act. For a uniform gravitational field, it coincides with the centre of mass.

Formalization – Rotational Equilibrium

For a body in equilibrium:

- Translational equilibrium: $\sum \vec{F} = 0$
- Rotational equilibrium: $\sum \tau = 0$ about any point

When taking moments, choose a pivot to simplify calculations (eliminate unknown forces passing through that point).

Moment of inertia I is the rotational analogue of mass. For a point mass m at distance r from axis: $I = mr^2$. For continuous bodies: $I = \int r^2 dm$. Common shapes:

- Hoop about centre: $I = MR^2$
- Solid cylinder/disc about centre: $I = \frac{1}{2}MR^2$
- Solid sphere about centre: $I = \frac{2}{5}MR^2$
- Rod about centre: $I = \frac{1}{12}ML^2$
- Rod about end: $I = \frac{1}{3}ML^2$

Parallel axis theorem: $I = I_{\text{cm}} + Md^2$.

Formalization – Rotational Dynamics

Newton's second law for rotation:

$$\tau_{\text{net}} = I\alpha$$

where α is angular acceleration. Kinematic equations for constant α:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Formalization – Rotational Energy and Work

Work done by a torque: $W = \tau \Delta\theta$ (for constant torque). Rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

For a rolling object without slipping, total kinetic energy:

$$K_{\text{total}} = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega^2$$

with $v_{\text{cm}} = R\omega$.

Formalization – Angular Momentum

Angular momentum of a point mass: $\vec{L} = \vec{r} \times \vec{p}$. For a rigid body rotating about a fixed axis: $L = I\omega$. Conservation of angular momentum: If net external torque on a system is zero, total angular momentum remains constant.

$$\tau_{\text{ext}} = \frac{dL}{dt}$$

Micro-Testing – Quick Checks

1. A force of 10 N is applied perpendicularly to a spanner at a distance 0.2 m from the nut. What is the torque?

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3. A disc of mass 2 kg and radius 0.1 m has moment of inertia? $I = \frac{1}{2}MR^2 = 0.5 \times 2 \times 0.01 = 0.01\ \mathrm{kg\,m^2}$.
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4. A skater spins with arms out, then pulls arms in. What happens to angular speed? Increases (conservation of L, I decreases).
5. A rolling ball has total KE. What fraction is rotational if it's a solid sphere? $K_r/K_t = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \frac{I}{MR^2} = \frac{2/5}{1} = 0.4$, so rotational fraction $= 0.4/(1+0.4) = 2/7$? Actually total $K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2(1 + I/(MR^2))$, so fraction rotational $= \frac{I/(MR^2)}{1+I/(MR^2)} = \frac{2/5}{7/5} = 2/7$.

HCI 2025 H2 Physics Prelim Paper 1 Q4

A metre rule of mass 50 g is suspended horizontally from the ceiling by two springs at its ends. The spring at the 0.0 cm mark has spring constant k_1, while the spring at the 100 cm mark has spring constant k_2. The springs have the same length when they are unstretched. A 200 g mass is placed at the 25.0 cm mark. What is the ratio k_2/k_1 such that the ruler is horizontal?

- A 0.33
- B 0.43
- C 2.3
- D 3.0

HCI 2025 P1 Q4 – Solution

Let T_1 and T_2 be tensions in springs at 0 cm and 100 cm. For rotational equilibrium about centre of mass (at 50 cm): Clockwise moments = Anticlockwise moments. Weight of rule ($0.050g$) acts at centre $\rightarrow$ no moment about centre. The 0.200 kg mass at 25 cm produces clockwise moment about centre? Distance from centre = 25 cm = 0.25 m. So clockwise moment $= 0.200g \times 0.25$. Tensions: T_1 at 0 cm (distance 0.50 m from centre) produces anticlockwise moment $= T_1 \times 0.50$. T_2 at 100 cm (distance 0.50 m) produces clockwise moment $= T_2 \times 0.50$. For equilibrium: $T_2 \times 0.50 + 0.200g \times 0.25 = T_1 \times 0.50$. Also vertical equilibrium: $T_1 + T_2 = (0.050 + 0.200)g = 0.250g$. From first: $0.50(T_1 - T_2) = 0.200g \times 0.25 = 0.05g \Rightarrow T_1 - T_2 = 0.1g$. Solving: $T_1 = 0.175g$, $T_2 = 0.075g$. Springs obey $F = kx$, and unstretched lengths equal, so extension $x = F/k$. Since both springs have same unstretched length and must have same stretched length to keep ruler horizontal, the extensions must be equal: $x_1 = x_2 \Rightarrow \frac{T_1}{k_1} = \frac{T_2}{k_2}$. Thus $\frac{k_2}{k_1} = \frac{T_2}{T_1} = \frac{0.075g}{0.175g} = \frac{75}{175} = \frac{3}{7} \approx 0.4286 \approx 0.43$. Answer: **B**.

RI 2025 H2 Physics Prelim Paper 1 Q5

Which statement is correct?

- A An object with only a couple acting on it is in translational equilibrium.
- B An object with only a couple acting on it is in equilibrium.
- C The direction of the torque of a couple depends on where the pivot is.
- D A couple is an action-reaction pair of forces.

RI 2025 P1 Q5 – Solution

A: Correct – a couple produces no net force, so translational equilibrium holds. B: Not in equilibrium because there is a net torque, so rotational acceleration occurs. C: The torque of a couple is independent of pivot; it is simply $F \times d$ (perpendicular distance between forces). D: A couple consists of two forces acting on the same body, not on different bodies, so they are not an action-reaction pair (which would act on different bodies). Answer: **A**.

A wheel is pulled over a kerb of height 0.080 m by a horizontal force F. The weight of the wheel is 700 N and radius 0.60 m. At the instant shown, the wheel just loses contact with the ground at G.

(a) Show that $\theta = 30^\circ$.

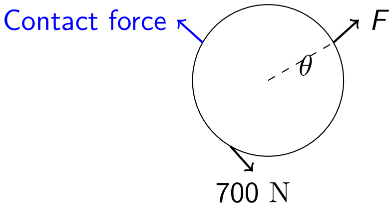
(b) On the diagram, draw an arrow to represent the contact force exerted on the wheel at P.

(c) Show that the minimum value of F is 190 N.

(d) Hence, determine the magnitude of the contact force at P.

From geometry: the line from centre to point of contact P makes angle θ with vertical. Vertical height from centre to kerb top = $0.60 - 0.08 = 0.52\ \mathrm{m}$. So $\cos\theta = \frac{0.52}{0.60} = 0.8667$, hence $\theta = 30^\circ$.

NJC 2025 P2 Q3 – Solution (b)



Contact force at P acts along the radius toward the centre (since no friction at point of contact when just losing contact with ground? Actually at the instant, the wheel is in contact only at P, and the kerb exerts a force perpendicular to the surface (normal force) which points toward the centre of the wheel. So arrow from P toward centre.

NJC 2025 P2 Q3 – Solution (c)

(c) Using principle of moments and taking moments about P:
$700 \times 0.60 \sin 30 = F(1.20 - 0.08)$ $F = 187.5 \approx 188\ \mathrm{N} = 190\ \mathrm{N}$.

NJC 2025 P2 Q3 – Solution (d)

Contact force at P is the resultant of normal and friction. Using vertical equilibrium: $R_y = 700\ \mathrm{N}$. Horizontal equilibrium: $R_x = F = 190\ \mathrm{N}$. So magnitude $R = \sqrt{700^2 + 190^2} = \sqrt{490000 + 36100} = \sqrt{526100} \approx 725\ \mathrm{N}$.

HCI 2025 H2 Physics Prelim Paper 2 Q2

A cantilever is set up on a rough table using a rigid uniform metre rule of mass 0.11 kg, a 1.5 kg block and a 5.0 g mass as shown. Determine the maximum number of 5.0 g masses that can be stacked above point X such that the cantilever does not topple.

HCI 2025 P2 Q2 – Solution

Let n be number of 5.0 g masses. Take moments about the edge of the table (the pivot point). The metre rule extends 0.50 m over the edge? The diagram shows the rule has the block at the left end, and masses at X near the right end. We need details from the solution. From HCI P2 Ans.pdf page 2: Anticlockwise moments: $(1.5g)(0.05)$? Actually they write: $(1.5g)(0.05) = (0.11g)(0.40) + n(0.005g)(0.80)$. Solving gives $n = 7$. Thus maximum number is 7.

RI 2025 H2 Physics Prelim Paper 2 Q2

A cantilever is set up using a rigid uniform metre rule of mass 0.11 kg, a 1.5 kg block and a 5.0 g mass. (a) Determine the maximum number of 5.0 g masses that can be stacked above point X such that the cantilever does not topple. (b) The structure is modified by adding an inextensible string that passes over a frictionless pulley with its ends tied to the 1.5 kg block and to the centre of the metre rule. State and explain how this modification will affect your answer in (a).

RI 2025 P2 Q2 – Solution (a)

Similar to HCI Q2, the solution yields $n = 7$. (Check RI P2 Ans.pdf page 1: they get 7.)

RI 2025 P2 Q2 – Solution (b)

Solution to be added.

Variation 1 – Seesaw Balance Difficulty: 2/10

A 3.0 m long uniform plank of mass 20 kg is pivoted at its centre. A 30 kg child sits at one end. Where must a 40 kg child sit to balance?

Variation 1 – Solution

Let distance from pivot for 40 kg child be d. Taking moments about pivot: $30g \times 1.5 = 40g \times d$ (plank's weight acts at pivot, no moment). $d = (30 \times 1.5)/40 = 45/40 = 1.125\ \mathrm{m}$ from pivot on opposite side.

A uniform ladder of length 5.0 m and mass 15 kg rests against a smooth vertical wall and rough horizontal ground ($\mu = 0.40$). The ladder makes 60° with the ground. How far up the ladder can a 70 kg person climb before the ladder slips?

Variation 2 – Solution

Forces: weight of ladder W_l at centre, person W_p at distance x from bottom, normal at wall N_w horizontal, normal at ground N_g vertical, friction $f = \mu N_g$ horizontal. Vertical equilibrium: $N_g = W_l + W_p = (15 + 70)g = 85g$. Horizontal: $f = N_w$. Moments about bottom: $N_w \times 5 \sin 60^\circ = W_l \times 2.5 \cos 60^\circ + W_p \times x \cos 60^\circ$.
$N_w = \mu N_g = 0.4 \times 85g = 34g$. Left side: $34g \times 5 \times 0.8660 = 34g \times 4.33 = 147.22g$.
Right side: $15g \times 2.5 \times 0.5 + 70g \times x \times 0.5 = 18.75g + 35gx$. Thus
$147.22g = 18.75g + 35gx \Rightarrow 128.47 = 35x \Rightarrow x \approx 3.67\ \mathrm{m}$ from bottom.

Variation 3 – Hinged Beam Difficulty: 4/10

A uniform beam of mass 10 kg and length 4.0 m is hinged at one end and supported by a cable at the other end making 30° with the beam. Find the tension in the cable.

Variation 3 – Solution

Forces: weight $W = 10g$ at centre, tension T at end at 30° to beam (so vertical component $T \sin 30$, horizontal $T \cos 30$). Hinge reaction R at pivot. Take moments about hinge: $W \times 2 \cos \theta_{\text{beam}}$? The beam is horizontal? Assume horizontal. Weight moment $= 10g \times 2$ (since weight at centre, distance 2 m). Tension moment: vertical component $T \sin 30$ acts at end, distance 4 m, so anticlockwise moment $= T \sin 30 \times 4$. Horizontal component has zero moment about hinge. Equilibrium: $4T \sin 30 = 20g \Rightarrow 4T \times 0.5 = 20g \Rightarrow 2T = 20g \Rightarrow T = 10g = 98.1\ \mathrm{N}$.

A shelf of weight 50 N and length 1.2 m is attached to a wall by a hinge at one end and a supporting wire at the other end at 45° to the horizontal. Find the tension in the wire.

Variation 4 – Solution

Take moments about hinge: weight 50 N acts at centre (0.6 m from hinge). Tension T at end (1.2 m) has vertical component $T \sin 45$.
Moment: $T \sin 45 \times 1.2 = 50 \times 0.6 = 30$.
$T \times 0.7071 \times 1.2 = 30 \Rightarrow 0.8485\,T = 30 \Rightarrow T \approx 35.4\ \mathrm{N}$.

Variation 5 – Flywheel Acceleration Difficulty: 4/10

A flywheel of moment of inertia 0.5 $\mathrm{kg\,m^2}$ is subjected to a constant torque of 2.0 Nm. Starting from rest, find its angular velocity after 5.0 s and the number of revolutions turned.

Variation 5 – Solution

Angular acceleration $\alpha = \tau / I = 2.0/0.5 = 4.0\ \mathrm{rad/s^2}$. After 5 s: $\omega = \alpha t = 4 \times 5 = 20\ \mathrm{rad/s}$. Angular displacement $\theta = \frac{1}{2}\alpha t^2 = 0.5 \times 4 \times 25 = 50\ \mathrm{rad}$. Revolutions $= 50/(2\pi) \approx 7.96$.

Variation 6 – Atwood Machine with Massive Pulley
Difficulty: 6/10

Two masses $m_1 = 2\ \mathrm{kg}$ and $m_2 = 3\ \mathrm{kg}$ hang over a pulley of mass $1\ \mathrm{kg}$ and radius $0.1\ \mathrm{m}$ (disc, $I = \frac{1}{2}MR^2$). Find the acceleration of the masses.

Variation 6 – Solution

Let a be acceleration of masses. For m_1: $T_1 - m_1 g = m_1 a$ upward? Actually set sign convention: let m_2 go down, m_1 up. For m_2: $m_2 g - T_2 = m_2 a$. For pulley: $(T_2 - T_1)R = I\alpha$, with $\alpha = a/R$. $I = \frac{1}{2}MR^2 = 0.5 \times 1 \times 0.01 = 0.005\ \mathrm{kg\,m^2}$. So $(T_2 - T_1)R = 0.005 \times a/R = 0.005a/R$. Multiply by R: $T_2 - T_1 = 0.005a/R^2 = 0.005a/0.01 = 0.5a$. Now solve: $T_1 = m_1(g + a)$, $T_2 = m_2(g - a)$. Substitute: $m_2(g - a) - m_1(g + a) = 0.5a$. $3(9.81 - a) - 2(9.81 + a) = 0.5a \Rightarrow 29.43 - 3a - 19.62 - 2a = 0.5a \Rightarrow 9.81 - 5a = 0.5a \Rightarrow 9.81 = 5.5a \Rightarrow a \approx 1.78\ \mathrm{m/s^2}$.

A uniform rod of length $L = 1.0\ \mathrm{m}$ and mass $2\ \mathrm{kg}$ is pivoted at one end and released from horizontal. Find its angular velocity when it becomes vertical.

Variation 7 – Solution

Moment of inertia about end: $I = \frac{1}{3}ML^2 = \frac{1}{3} \times 2 \times 1^2 = \frac{2}{3}\ \mathrm{kg\,m^2}$. Centre of mass falls a distance $L/2 = 0.5\ \mathrm{m}$. Energy conservation: $Mg\frac{L}{2} = \frac{1}{2}I\omega^2$. $2 \times 9.81 \times 0.5 = 9.81 = \frac{1}{2} \times \frac{2}{3} \times \omega^2 = \frac{1}{3}\omega^2$. $\omega^2 = 29.43$, $\omega \approx 5.43\ \mathrm{rad/s}$.

Variation 8 – Skater Spins   Difficulty: 3/10

A skater spins at 2.0 $\mathrm{rev/s}$ with arms out, moment of inertia 3.0 $\mathrm{kg\,m^2}$. She pulls arms in, reducing I to 1.2 $\mathrm{kg\,m^2}$. Find her new angular speed.

Variation 8 – Solution

Conservation of angular momentum: $I_1\omega_1 = I_2\omega_2$.
$\omega_1 = 2 \times 2\pi = 4\pi \text{ rad/s}$.
$\omega_2 = I_1\omega_1/I_2 = 3 \times 4\pi/1.2 = 12\pi/1.2 = 10\pi \text{ rad/s} = 5 \text{ rev/s}$.

Variation 9 – Collision with Pivot Difficulty: 5/10

A door (width 1.0 m, mass 15 kg, hinged at one end) is at rest. A ball of mass 0.5 kg moving at 10 $\mathrm{m/s}$ strikes the door perpendicularly at its centre and sticks. Find the angular speed of the door after impact.

Variation 9 – Solution

Moment of inertia of door about hinges:
$I_d = \frac{1}{3}ML^2 = \frac{1}{3} \times 15 \times 1^2 = 5\ \mathrm{kg\,m^2}$. Ball's angular momentum about hinges before impact: $L_{\text{ball}} = mvr = 0.5 \times 10 \times 0.5 = 2.5\ \mathrm{kg\,m^2/s}$. After sticking, total $I = I_d + mr^2 = 5 + 0.5 \times 0.25 = 5.125\ \mathrm{kg\,m^2}$. Angular momentum conserved: $2.5 = 5.125\,\omega \Rightarrow \omega \approx 0.488\ \mathrm{rad/s}$.

Variation 10 – Rolling Down Incline Difficulty: 4/10

A solid sphere (mass $2\ \mathrm{kg}$, radius $0.1\ \mathrm{m}$) rolls without slipping down a 30° incline of length $2\ \mathrm{m}$. Find its speed at the bottom.

Variation 10 – Solution

Energy conservation: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}mR^2$, $\omega = v/R$.
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}mR^2)(v^2/R^2) = \frac{1}{2}mv^2(1+\frac{2}{5}) = \frac{1}{2}mv^2 \times \frac{7}{5}$.
$gh = \frac{7}{10}v^2 \Rightarrow v = \sqrt{\frac{10}{7}gh}$. $h = 2\sin 30^\circ = 1\ \mathrm{m}$.
$v = \sqrt{\frac{10}{7} \times 9.81} \approx \sqrt{14.014} \approx 3.74\ \mathrm{m/s}$.

Variation 11 – Yo-yo Difficulty: 5/10

A yo-yo of mass 0.1 kg and moment of inertia $I = 5 \times 10^{-5}\ \mathrm{kg\,m^2}$ has string wrapped around axle of radius 0.01 m. It is released from rest. Find acceleration.

Variation 11 – Solution

Forces: $mg - T = ma$. Torque: $Tr = I\alpha$, with $a = \alpha r$. So $T = Ia/r^2$.
Substitute: $mg - Ia/r^2 = ma \Rightarrow mg = a(m + I/r^2)$.
$a = \frac{mg}{m+I/r^2} = \frac{0.1\times9.81}{0.1+5\times10^{-5}/10^{-4}} = \frac{0.981}{0.1+0.5} = \frac{0.981}{0.6} \approx 1.635\ \mathrm{m/s^2}$.

Challenge 1 – Pivoted Rod with Hanging Mass Difficulty: 7/10

A uniform rod of mass M and length L is pivoted at one end. A mass m is attached to the other end. The rod is released from horizontal. Find the angular speed when vertical.

Challenge 1 – Solution

Moment of inertia: $I = \frac{1}{3}ML^2 + mL^2$. Centre of mass of rod at $L/2$, mass m at L. Potential energy change: $\Delta U = Mg\frac{L}{2} + mgL = gL\left(\frac{M}{2} + m\right)$. Energy: $\Delta U = \frac{1}{2}I\omega^2$. $\omega^2 = \frac{2gL\left(\frac{M}{2}+m\right)}{\frac{1}{3}ML^2+mL^2} = \frac{2g\left(\frac{M}{2}+m\right)}{L\left(\frac{1}{3}M+m\right)}$.

Challenge 2 – Rotating Collision Difficulty: 8/10

A uniform rod of mass M and length L is pivoted at its centre. A particle of mass m moving with speed v strikes one end perpendicularly and sticks. Find angular speed of system after collision.

Challenge 2 – Solution

Before collision, angular momentum about pivot: $L_i = mv(L/2)$ (since particle strikes at end, distance $L/2$ from centre). After sticking, total moment of inertia: $I = \frac{1}{12}ML^2 + m(L/2)^2 = \frac{1}{12}ML^2 + \frac{1}{4}mL^2$. Angular momentum conserved: $mvL/2 = I\omega$. $\omega = \frac{mvL/2}{\frac{1}{12}ML^2+\frac{1}{4}mL^2} = \frac{mv/2}{L\left(\frac{1}{12}M+\frac{1}{4}m\right)}$.

Challenge 3 – Rolling on a Loop Difficulty: 9/10

A small solid sphere rolls without slipping from rest at height h on a track that leads into a vertical loop of radius R. Find the minimum h so that the sphere completes the loop without losing contact.

Challenge 3 – Solution

At top of loop, speed must satisfy $mg + N = mv^2/R$, with $N \geq 0$ so $v^2 \geq gR$. For rolling sphere, $v = \omega R$, $I = \frac{2}{5}mR^2$. Energy from start to top:
$mg(h - 2R) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot (v^2/R^2) = \frac{1}{2}mv^2(1 + \frac{2}{5}) = \frac{7}{10}mv^2$.
Thus $g(h - 2R) = \frac{7}{10}v^2 \geq \frac{7}{10}gR \Rightarrow h - 2R \geq \frac{7}{10}R \Rightarrow h \geq 2.7R$.

Challenge 4 – Gyroscope Precession Difficulty: 9/10

A gyroscope consists of a disc of mass m and radius r spinning at angular speed ω on an axle of length L pivoted at one end. Find the precession angular speed Ω.

Challenge 4 – Solution

Torque due to weight: $\tau = mgL$ (horizontal). Angular momentum of disc: $L_{\text{spin}} = I\omega = \frac{1}{2}mr^2\omega$. Precession rate $\Omega = \tau / L_{\text{spin}} = \frac{mgL}{\frac{1}{2}mr^2\omega} = \frac{2gL}{r^2\omega}$.

Challenge 5 – Falling Chimney Difficulty: 8/10

A tall uniform chimney of height H and mass M is hinged at the bottom and falls over. Find the angular speed when it makes angle θ with the vertical, and determine the point where it might break (stress maximum).

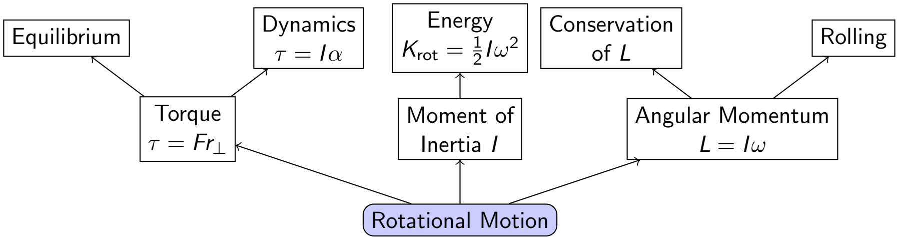
Challenge 5 – Solution

Moment of inertia about bottom: $I = \frac{1}{3}MH^2$. Centre of mass falls distance $\frac{H}{2}(1-\cos\theta)$. Energy: $Mg\frac{H}{2}(1-\cos\theta) = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{1}{3}MH^2\omega^2 \Rightarrow \omega^2 = \frac{3g}{H}(1-\cos\theta)$. The bending moment is maximum at a point about 1/3 from bottom, which is where chimney often breaks.

End-of-Session Concept Recap

- Torque = force × perpendicular distance from pivot.
- Rotational equilibrium: sum of torques = 0.
- Moment of inertia I depends on mass distribution.
- Rotational dynamics: $\tau_{\text{net}} = I\alpha$.
- Rotational kinetic energy: $K_{\text{rot}} = \frac{1}{2}I\omega^2$.
- Angular momentum $L = I\omega$, conserved if no external torque.
- Rolling without slipping: $v = \omega R$, total KE = translational + rotational.
- Applications: seesaws, ladders, flywheels, spinning skaters, rolling objects.

Mind Map



Link to linear motion: analogous quantities – force/torque, mass/moment of inertia, linear momentum/angular momentum.