H2 Physics
Potential Energy and Energy Conservation

R.F.H.

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Brain Dump

What do you know about Potential Energy and Energy Conservation?

Brain Dump (CONT'D)

What do you know about Potential Energy and Energy Conservation?

- Gravitational potential energy near Earth: $E_p = mgh$
- Gravitational potential energy in space: $U = -\dfrac{GMm}{r}$
- Elastic potential energy: $E_e = \dfrac{1}{2}kx^2$
- Electric potential energy: $U_E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r}$
- Energy stored in a capacitor: $U = \dfrac{1}{2}QV = \dfrac{1}{2}CV^2$
- Work-energy theorem: $W_{\text{net}} = \Delta E_k$
- Conservation of mechanical energy: $E_k + E_p = \text{constant}$ (if no non-conservative forces)
- Power: $P = \dfrac{dE}{dt} = Fv$ (for constant force)
- Efficiency: $\eta = \dfrac{\text{useful energy output}}{\text{total energy input}}$

Math Checklist

Before tackling Energy, ensure you are comfortable with:

- Solving quadratic equations
- Algebraic manipulation of formulas
- Integration (area under force-displacement graph)
- Differentiation (rate of change of energy)

- Trigonometric functions
- Logarithms and exponentials (for decay problems)
- Vector dot product (work $= \vec{F} \cdot \vec{s}$)
- Units and conversions (J, eV, kWh)

Building Intuition – Real-world Applications

- **Roller coasters**: conversion between gravitational potential energy and kinetic energy.
- **Pendulum**: energy oscillates between kinetic and potential.
- **Bow and arrow**: elastic potential energy stored in limbs converted to kinetic energy of arrow.
- **Hydroelectric dam**: gravitational potential energy of water converted to electrical energy.
- **Satellite orbits**: total energy $E = -\frac{GMm}{2r}$ (negative indicates bound system).
- **Capacitors**: energy stored in electric field.
- **Nuclear reactions**: mass-energy equivalence $E = mc^2$.

Formalization – Work and Energy

Work Done by a Constant Force

$W = Fs \cos \theta$, where θ is angle between force and displacement.

Kinetic Energy

$E_k = \frac{1}{2} mv^2$

Work-Energy Theorem

$W_{\text{net}} = \Delta E_k$ (net work done on an object equals its change in kinetic energy)

Conservation of Mechanical Energy

If only conservative forces (gravity, spring force, electric force) do work,

$$E_k + E_p = \text{constant}$$

Formalization – Potential Energy Types

- **Gravitational potential energy (near Earth)**: $E_p = mgh$ (reference level at $h = 0$)

- **Gravitational potential energy (general)**: $U = -\dfrac{GMm}{r}$ (zero at infinity)

- **Elastic potential energy**: $E_e = \dfrac{1}{2}kx^2$ (zero at natural length)

- **Electric potential energy (point charges)**: $U_E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r}$ (zero at infinity)

- **Energy stored in a capacitor**: $U = \dfrac{1}{2}QV = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{Q^2}{C}$

Formalization – Power and Efficiency

Power

Rate of energy transfer: $P = \dfrac{dE}{dt}$. For constant force and velocity:
$P = Fv\cos\theta$.

Efficiency

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Energy losses often due to friction, heat, sound, etc.

Micro-Testing – Quick Checks

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Two frictionless trolleys move along the same straight line towards one another. Masses and velocities before collision are shown. The trolleys collide and stick together. What is the final kinetic energy of the trolleys after the collision?

(Diagram: 5.0 kg at 4.0 m/s right, 2.0 kg at 3.0 m/s left)

- Ⓐ 0.71 J
- Ⓑ 14 J
- Ⓒ 31 J
- Ⓓ 35 J

Conservation of momentum (right positive):

$$(5.0)(4.0) + (2.0)(-3.0) = (5.0 + 2.0)v$$

$$20.0 - 6.0 = 7.0v \Rightarrow v = 2.0 \ \mathrm{m/s}$$

Final kinetic energy:

$$E_k = \frac{1}{2}(7.0)(2.0)^2 = 0.5 \times 7.0 \times 4 = 14 \ \mathrm{J}$$

Answer: **B**.

A turbine at a hydroelectric power station is situated at a vertical distance 30 m below the surface of a large lake. Water passes through the turbine at 5.7 m^3/s. Overall efficiency 90%, density of water 1000 kg/m^3. What is the useful power output?

- **A** 0.15 MW
- **B** 1.5 MW
- **C** 1.7 MW
- **D** 90 MW

Mass flow rate: $\dot{m} = \rho \times$ volume flow rate $= 1000 \times 5.7 = 5700 \ \text{kg/s}$.

Power available from falling water: $P_{\text{in}} = \dot{m}gh = 5700 \times 9.81 \times 30$.

$P_{\text{in}} = 5700 \times 294.3 = 1\,677\,510 \ \text{W} \approx 1.68 \ \text{MW}$. Useful power $= 0.90 \times 1.68 = 1.51 \ \text{MW} \approx 1.5 \ \text{MW}$. Answer: **B**.

A huge block of ice at $0°C$ with a hollow in its top surface. A mass of 160 g of water at $100°C$ is poured into the hollow. Specific heat capacity of water $4.20 \text{ kJ kg}^{-1}\text{K}^{-1}$, latent heat of fusion of ice 336 kJ kg^{-1}. After thermal equilibrium, what is the total mass of water in the hollow?

- **Ⓐ** 100 g
- **Ⓑ** 200 g
- **Ⓒ** 260 g
- **Ⓓ** 360 g

Heat lost by hot water cooling to $0^\circ\mathrm{C}$:
$Q = mc\Delta T = 0.160 \times 4200 \times 100 = 67200$ J. This heat melts ice: mass melted $= \frac{Q}{L} = \frac{67200}{336000} = 0.200$ kg $= 200$ g. Total water in hollow $=$ original 160 g $+$ melted 200 g $= 360$ g. Answer: **D**.

An alpha particle with kinetic energy 9.0×10^{-13} J approaches a stationary gold nucleus (79 protons). Find the closest possible distance of approach.

- Ⓐ 2.5×10^{-16} m
- Ⓑ 2.0×10^{-14} m
- Ⓒ 4.0×10^{-14} m
- Ⓓ 2.0×10^{-7} m

At closest approach, kinetic energy is converted to electric potential energy:

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\frac{Q_\alpha Q_{\text{Au}}}{r}$$

$Q_\alpha = 2e$, $Q_{\text{Au}} = 79e$.

$$9.0 \times 10^{-13} = \frac{(8.99 \times 10^9)(2 \times 1.60 \times 10^{-19})(79 \times 1.60 \times 10^{-19})}{r}$$

Numerator: $8.99 \times 10^9 \times 3.2 \times 10^{-19} \times 1.264 \times 10^{-17} =$
$8.99 \times 10^9 \times 4.0448 \times 10^{-36} = 3.637 \times 10^{-26}$. Thus
$r = \frac{3.637 \times 10^{-26}}{9.0 \times 10^{-13}} = 4.04 \times 10^{-14}$ m $\approx 4.0 \times 10^{-14}$ m. Answer: **C**.

A wire is stretched elastically by a force of 200 N, causing an extension of 2.00 mm. The force is gradually increased to 250 N, wire remains elastic. What is the work done in stretching from 200 N to 250 N?

- **A** 0.113 J
- **B** 0.225 J
- **C** 113 J
- **D** 225 J

Spring constant $k = \frac{F}{x} = \frac{200}{0.002} = 100000 \text{ N/m}$. Extension at 250 N: $x_2 = \frac{250}{100000} = 0.0025 \text{ m}$. Work done = area under force-extension graph (trapezoid):

$$W = \frac{1}{2}(200+250)(0.0025-0.002) = 0.5 \times 450 \times 0.0005 = 0.1125 \text{ J} \approx 0.113 \text{ J}$$

Answer: **A**.

The force F required to extend a spring of unstretched length x_0 to length x is measured. When tension is T_1, length is x_1; when tension T_2, length x_2. What is the work done to stretch from x_1 to x_2?

- (A) $\frac{1}{2}T_2(x_2 - x_0)$
- (B) $\frac{1}{2}(T_1 + T_2)(x_2 - x_1)$
- (C) $\frac{1}{2}(T_1 + T_2)(x_2 + x_1 - 2x_0)$
- (D) $\frac{1}{2}(T_1 + T_2)(x_2 - x_1 - 2x_0)$

Work done = area under force-extension graph. Extension is $(x - x_0)$. At x_1, extension $e_1 = x_1 - x_0$; at x_2, extension $e_2 = x_2 - x_0$. Work = area of trapezoid under force vs extension:

$$W = \frac{1}{2}(T_1 + T_2)(e_2 - e_1) = \frac{1}{2}(T_1 + T_2)[(x_2 - x_0) - (x_1 - x_0)] = \frac{1}{2}(T_1 + T_2)(x_2 -$$

Answer: **B**.

A driving force of 250 N is needed for a car of mass 900 kg to travel along a level road at constant speed 24 m/s. What power is required to maintain this speed when moving up a slope that rises 1.0 m for every 12 m of travel?

- **A** 6.8 kW
- **B** 12 kW
- **C** 18 kW
- **D** 24 kW

On slope, component of weight down slope $= mg \sin\theta$, with $\sin\theta = \frac{1}{12}$.
Total force required
$= 250 + 900 \times 9.81 \times \frac{1}{12} = 250 + 8829/12 = 250 + 735.75 = 985.75$ N.
Power $= Fv = 985.75 \times 24 = 23658$ W ≈ 23.7 kW. Closest answer: **D** (24 kW).

A pellet of mass 8.00×10^{-3} kg is projected at angle θ above horizontal with speed u. It reaches maximum height 10.0 m and speed 5.00 m/s at maximum height. Air resistance negligible.

(a)(i) Show $u = 14.9$ m/s using energy conservation.

(a)(ii) Calculate θ.

At max height, vertical velocity zero, so speed $= u \cos \theta = 5.00$. Energy conservation: loss in KE = gain in GPE.

$$\frac{1}{2} m u^2 - \frac{1}{2} m (5.00)^2 = mg(10.0)$$

Cancel m: $\frac{1}{2} u^2 - \frac{1}{2}(25) = 98.1$

$0.5 u^2 = 98.1 + 12.5 = 110.6 \Rightarrow u^2 = 221.2 \Rightarrow u \approx 14.87 \text{ m/s} \approx 14.9 \text{ m/s}$

From $u\cos\theta = 5.00$, $\cos\theta = \frac{5.00}{14.87} \approx 0.3362$, $\theta \approx 70.4°$.

A bullet of mass 2.0 g is fired horizontally into a block of wood of mass 600 g suspended by strings. The bullet embeds, and the block rises 8.6 cm.

(b)(i) Show speed of block+bullet just after impact is 1.3 m/s.

(b)(ii) Find speed of bullet before impact.

After impact, kinetic energy converts to gravitational potential energy:

$$\frac{1}{2}(0.602)v^2 = (0.602)g(0.086)$$

Cancel 0.602: $\frac{1}{2}v^2 = 9.81 \times 0.086 = 0.84366$

$$v^2 = 1.68732 \Rightarrow v \approx 1.299 \text{ m/s} \approx 1.3 \text{ m/s}$$

Conservation of momentum during collision:

$$0.002\,u = 0.602 \times 1.299$$

$$u = \frac{0.602 \times 1.299}{0.002} = \frac{0.782}{0.002} = 391 \text{ m/s}$$

A ski lift uses 48 two-person chairs, each mass 80 kg, moving at 2.5 $\mathrm{m/s}$ up a height of 300 m over a distance 900 m. Average skier mass 75 kg. Calculate mechanical power required.

Number of skiers per chair = 2, so total skiers = $48 \times 2 = 96$. Total mass of skiers = $96 \times 75 = 7200$ kg. Total mass of chairs = $48 \times 80 = 3840$ kg. Total mass = 11040 kg. Time to travel 900 m at 2.5 m/s: $t = 900/2.5 = 360$ s. Power = rate of gain of gravitational potential energy:

$$P = \frac{mgh}{t} = \frac{11040 \times 9.81 \times 300}{360}$$

$11040 \times 9.81 = 108302.4$, times 300 = 32490720, divide by $360 = 90252$ W ≈ 90.3 kW.

A yoke of mass 0.30 kg moves with SHM, amplitude 0.080 m, period 0.40 s. Determine maximum speed and maximum acceleration. Then sketch net force vs time. (Energy not explicitly asked, but we can extract energy.)

Maximum speed $v_0 = \omega r = \frac{2\pi}{0.40} \times 0.080 = 15.708 \times 0.08 = 1.2566 \ \mathrm{m/s}$.
Maximum kinetic energy
$= \frac{1}{2}mv_0^2 = 0.5 \times 0.30 \times (1.2566)^2 = 0.15 \times 1.579 = 0.237 \ \mathrm{J}$. This equals
total energy of oscillation.

The Earth radius $R = 6.4 \times 10^6$ m, mass $M = 6.0 \times 10^{24}$ kg. A meteorite falls from rest at a great distance. Find its speed when at distance $3R$ from Earth's centre. (Use graph of ϕ vs x.)

Gravitational potential at infinity $= 0$, at $3R$: from graph, $\phi \approx -2.1 \times 10^7$ J/kg. Conservation of energy:
$0 = \frac{1}{2}v^2 + \phi \Rightarrow v = \sqrt{-2\phi} = \sqrt{4.2 \times 10^7} \approx 6481$ m/s.

Planet Z has density ρ and radius r. Show escape velocity $v = \sqrt{\frac{8}{3}G\pi\rho r^2}$. Given $\rho = 5500 \ \mathrm{kg/m^3}$, $r = 413 \ \mathrm{km}$, calculate v. Then find temperature of atmosphere if $v = c_{\mathrm{rms}}$.

Escape velocity: $\frac{1}{2}mv^2 = \frac{GMm}{r}$, with $M = \frac{4}{3}\pi r^3 \rho$. Thus
$v^2 = \frac{2G}{r} \cdot \frac{4}{3}\pi r^3 \rho = \frac{8}{3}G\pi\rho r^2$, so $v = \sqrt{\frac{8}{3}G\pi\rho r^2}$.

$v = \sqrt{\frac{8}{3}(6.67 \times 10^{-11})\pi(5500)(413 \times 10^3)^2}$. First compute
$r^2 = (4.13 \times 10^5)^2 = 1.706 \times 10^{11}$.
$\rho r^2 = 5500 \times 1.706 \times 10^{11} = 9.383 \times 10^{14}$.
$G\pi\rho r^2 = 6.67 \times 10^{-11} \times \pi \times 9.383 \times 10^{14} =$
$6.67 \times 10^{-11} \times 2.947 \times 10^{15} = 1.966 \times 10^5$. $\frac{8}{3}$ of that
$= 2.6667 \times 1.966 \times 10^5 = 5.243 \times 10^5$. $v = \sqrt{5.243 \times 10^5} \approx 724$ m/s.

$c_{\mathsf{rms}} = v = \sqrt{\frac{3RT}{M}}$ where M is molar mass ($40 \mathrm{\ g/mol} = 0.040 \mathrm{\ kg/mol}$).

$T = \frac{Mv^2}{3R} = \frac{0.040 \times (724)^2}{3 \times 8.31} = \frac{0.040 \times 524176}{24.93} = \frac{20967}{24.93} \approx 841 \mathrm{\ K}$.

A ball is thrown at 25 m/s, 30°. Find max height, and at that height express KE and PE in terms of initial KE K.

$u_y = 25 \sin 30° = 12.5 \text{ m/s}$. Max height $h = \frac{u_y^2}{2g} = \frac{156.25}{19.62} = 7.96 \text{ m}$. At top, speed $= u_x = 25 \cos 30° = 21.65 \text{ m/s}$, so KE $= \frac{1}{2}m(21.65)^2 = \frac{1}{2}mu^2 \left(\frac{21.65}{25}\right)^2 = K \times 0.75$. PE $= mgh = mg(7.96) = \frac{1}{2}mu^2 \times \frac{2gh}{u^2} = K \times \frac{2 \times 9.81 \times 7.96}{625} = K \times \frac{156.2}{625} = K \times 0.25$. So KE $= 0.75K$, PE $= 0.25K$.

An archer fires an arrow at 52 $\mathrm{m/s}$, 15° from height 1.5 m. It hits a tree at height 8.0 m. Find KE just before impact and distance to tree.

Energy conservation: $\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mg(8.0 - 1.5)$.
$\frac{1}{2}v_f^2 = \frac{1}{2}(52)^2 - 9.81 \times 6.5 = 1352 - 63.765 = 1288.235$. $v_f^2 = 2576.47$,
$v_f \approx 50.76 \text{ m/s}$. KE $= \frac{1}{2}m(50.76)^2$. With $m = 0.032 \text{ kg}$, KE
$= 0.5 \times 0.032 \times 2576.47 = 0.016 \times 2576.47 = 41.22 \text{ J}$.

$u_x = 52\cos 15° \approx 50.24 \ \mathrm{m/s}$, $u_y = 52\sin 15° \approx 13.46 \ \mathrm{m/s}$. Vertical motion: $s_y = u_y t - \frac{1}{2}gt^2$, $s_y = 6.5 \ \mathrm{m}$.
$6.5 = 13.46t - 4.905t^2 \Rightarrow 4.905t^2 - 13.46t + 6.5 = 0$. Solve:
$t = \frac{13.46 \pm \sqrt{181.17 - 127.53}}{9.81} = \frac{13.46 \pm 7.324}{9.81}$. $t = 0.626 \ \mathrm{s}$ (on way up) or $2.12 \ \mathrm{s}$ (down). Use $t = 0.626$. $x = u_x t = 50.24 \times 0.626 \approx 31.4 \ \mathrm{m}$.

A stone of mass 0.5 kg is dropped from a height of 20 m. Find its speed just before hitting the ground.

Variation 1 – Solution

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 20} = \sqrt{392.4} \approx 19.8 \text{ m/s}.$$

A roller coaster car starts from rest at height h above the bottom of a frictionless loop of radius R. What minimum h is needed for the car to just make it around the loop?

At top of loop, need $v \geq \sqrt{gR}$. Energy conservation:
$mgh = mg(2R) + \frac{1}{2}mgR = \frac{5}{2}mgR$, so $h = \frac{5}{2}R$.

A satellite of mass 500 kg orbits Earth at a height of 300 km. Earth mass $M = 6.0 \times 10^{24} \text{ kg}$, radius $R = 6.37 \times 10^6 \text{ m}$, $G = 6.67 \times 10^{-11}$. Find its total mechanical energy.

Variation 3 – Solution

Orbital radius $r = R + h = 6.37 \times 10^6 + 3.0 \times 10^5 = 6.67 \times 10^6$ m. Total energy $E = -\frac{GMm}{2r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 500}{2 \times 6.67 \times 10^6}$.

$GMm = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 500 = 2.001 \times 10^{17}$. Divide by $2r = 1.334 \times 10^7$: $E = -1.5 \times 10^{10}$ J.

A spring with $k = 800 \ \mathrm{N/m}$ is compressed by $0.15 \ \mathrm{m}$. How much energy is stored? If a $0.2 \ \mathrm{kg}$ mass is placed against it and released, what speed does the mass have when the spring returns to natural length?

Variation 4 – Solution

$E_e = \frac{1}{2}kx^2 = 0.5 \times 800 \times 0.0225 = 9$ J.

$v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{18}{0.2}} = \sqrt{90} \approx 9.49$ m/s.

Two identical springs of constant $k = 300$ N/m are connected in parallel. A mass of 2 kg is attached and displaced 0.1 m from equilibrium. Find the total energy stored.

Variation 5 – Solution

Effective $k_{\text{eff}} = 2k = 600 \text{ N/m}$. Energy
$= \frac{1}{2} k_{\text{eff}} x^2 = 0.5 \times 600 \times 0.01 = 3 \text{ J}$.

Two point charges $Q_1 = +3$ μC and $Q_2 = -2$ μC are initially 0.5 m apart. How much work is required to separate them to 1.0 m?

Initial potential energy: $U_i = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_i} = 8.99 \times 10^9 \times \frac{(3\times10^{-6})(-2\times10^{-6})}{0.5} = 8.99 \times 10^9 \times \frac{-6\times10^{-12}}{0.5} = 8.99 \times 10^9 \times (-1.2 \times 10^{-11}) = -0.1079$ J. Final: $U_f = 8.99 \times 10^9 \times \frac{-6\times10^{-12}}{1.0} = -0.05394$ J. Work done by external force $= \Delta U = U_f - U_i = -0.05394 - (-0.1079) = 0.05396$ J.

A 100 μF capacitor is charged to 50 V. How much energy is stored? If discharged through a resistor, how much heat is dissipated?

$U = \frac{1}{2}CV^2 = 0.5 \times 100 \times 10^{-6} \times 2500 = 0.125$ J. All this energy becomes heat.

A force of 30 N pushes a box 5 m across a floor. How much work is done? If the force is applied at $40°$ to the horizontal, what is the work?

Variation 8 – Solution

$W = Fs = 30 \times 5 = 150$ J. With angle:
$W = Fs \cos 40^\circ = 150 \times 0.7660 = 114.9$ J.

A motor lifts a 200 kg crate at constant speed 1.5 m/s. What is the power output of the motor?

$P = Fv = mgv = 200 \times 9.81 \times 1.5 = 2943 \text{ W} \approx 2.94 \text{ kW}.$

An electric motor has an efficiency of 85%. It lifts a 50 kg mass through 10 m in 5 s. What electrical power is drawn?

Useful power $= \frac{mgh}{t} = \frac{50 \times 9.81 \times 10}{5} = \frac{4905}{5} = 981$ W. Electrical power $= \frac{981}{0.85} \approx 1154$ W.

A block starts from rest at height h above the bottom of a loop of radius R. The track has coefficient of kinetic friction μ on the horizontal approach only. Find h such that the block just makes it around the loop.

Speed needed at top: $v_{\text{top}} = \sqrt{gR}$. Energy from start to top:
$mgh = mg(2R) + \frac{1}{2}mgR + \mu mgd$, where d is horizontal distance before
loop. So $h = \frac{5}{2}R + \mu d$.

A mass attached to a string is rotated in a horizontal circle. The string is slowly pulled down through a hole, reducing radius. If initial radius r_0 and angular velocity ω_0, find final angular velocity when radius is r_f. (Energy not conserved because tension does work.)

Angular momentum $L = mr^2\omega$ is conserved (torque zero about centre). So $r^2\omega = $ constant, thus $\omega_f = \omega_0(r_0/r_f)^2$. Energy is not conserved because work is done pulling the string.

A spacecraft approaches a planet of mass M with speed v_0 (relative to planet) and impact parameter b. Derive the condition for it to gain energy from the encounter.

In planet's frame, spacecraft's speed is unchanged (elastic collision with planet), but direction changes. In Sun's frame, if planet is moving, the spacecraft can gain or lose energy. Maximum gain when spacecraft approaches from behind and exits ahead. Analysis uses conservation of energy and angular momentum in planet's frame, then transforming back.

Four equal positive charges q are placed at corners of a square of side a. Find the total electric potential energy of the system.

Challenge 4 – Solution

There are 6 pairs. For adjacent corners: distance a, energy per pair $= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}$, 4 such pairs. For diagonal pairs: distance $a\sqrt{2}$, energy per pair $= \frac{1}{4\pi\epsilon_0}\frac{q^2}{a\sqrt{2}}$, 2 such pairs. Total $U = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\left(4 + \frac{2}{\sqrt{2}}\right) = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\left(4 + \sqrt{2}\right)$.

A capacitor C is charged through a resistor R from a battery of emf V. Show that the energy supplied by the battery is twice the energy stored in the capacitor, and explain where the other half goes.

Energy from battery $= \int VI dt = V \int I dt = VQ = V(CV) = CV^2$. Energy stored in capacitor $= \frac{1}{2}CV^2$. The other half is dissipated as heat in the resistor. This can be shown by integrating I^2R over time.

End-of-Session Concept Recap

- Energy is a scalar quantity, conserved in isolated systems.
- Work done by conservative forces is path-independent and stored as potential energy.
- Gravitational potential energy: mgh (near Earth) or $-\frac{GMm}{r}$ (general).
- Elastic potential energy: $\frac{1}{2}kx^2$.
- Electric potential energy: $\frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$.
- Energy in capacitor: $\frac{1}{2}CV^2$.
- Power: rate of energy transfer, $P = Fv$ for constant force.
- Efficiency accounts for energy losses.

Mind Map
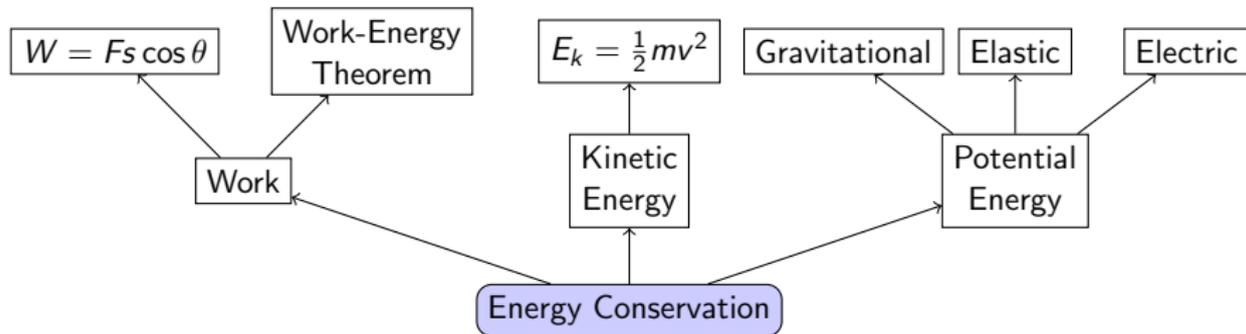
Link to dynamics: forces do work, changing energy. Link to thermal: energy dissipation. Link to circuits: energy in capacitors and batteries.