H2 Physics
Kinetic Energy and Work

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Brain Dump

What do you know about Kinetic Energy and Work?

Brain Dump (CONT'D)

What do you know about Kinetic Energy and Work?

- Definition of work: $W = \vec{F} \cdot \vec{s} = Fs\cos\theta$
- Work done by variable force: area under F-x graph
- Kinetic energy: $K = \frac{1}{2}mv^2$
- Work-energy theorem: $W_{\text{net}} = \Delta K$
- Conservative forces and potential energy
- Conservation of mechanical energy
- Power: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$
- Efficiency: $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{E_{\text{useful}}}{E_{\text{input}}}$
- Elastic potential energy: $U = \frac{1}{2}kx^2$
- Gravitational potential energy near Earth: $U = mgh$

Math Checklist

Before tackling Work and Energy, ensure you are comfortable with:

- Dot product of vectors
- Integration (area under curve)
- Solving quadratic equations
- Trigonometry: sin, cos, tan
- Differentiation (rate of change for power)
- Logarithms and exponentials (for variable forces)
- Unit conversions (J, eV, kWh)
- Percentage uncertainties

Building Intuition – Real-world Applications

- Car braking: work done by friction converts kinetic energy to heat.
- Roller coaster: conversion between gravitational potential energy and kinetic energy.
- Bow and arrow: work done by archer stored as elastic potential energy, then converted to kinetic energy.
- Hydroelectric dam: gravitational potential energy of water converted to kinetic energy, then to electrical energy.
- Wind turbine: kinetic energy of wind converted to rotational kinetic energy, then to electricity.
- Electric vehicles: battery electrical energy converted to kinetic energy via motors.

Formalization – Work

Work Done by a Constant Force

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

where θ is the angle between force and displacement.

Work Done by a Variable Force

For a force varying with position, work done = area under F-x graph:

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Work-Energy Theorem

The net work done on an object equals its change in kinetic energy:

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Formalization – Kinetic Energy

Kinetic Energy

Energy associated with motion:

$$K = \frac{1}{2}mv^2$$

Derivation: from $W = \int F\,dx = \int m\frac{dv}{dt}v\,dt = \int mv\,dv = \frac{1}{2}mv^2$.

Power

Rate of doing work:

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

For constant force and velocity, $P = Fv\cos\theta$.

Formalization – Potential Energy and Conservation

For conservative forces (gravity, spring), work done can be expressed as change in potential energy:

$$W_{\text{cons}} = -\Delta U$$

Then total mechanical energy $E = K + U$ is conserved if only conservative forces act:

$$K_i + U_i = K_f + U_f$$

Micro-Testing – Quick Checks

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2. A force of 10 N acts at 60° to the displacement of 5 m. How much work is done?

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2. A force of 10 N acts at 60° to the displacement of 5 m. How much work is done? $W = 10 \times 5 \times \cos 60^\circ = 50 \times 0.5 = 25\ \mathrm{J}$.
3. What is the power of a motor lifting a 100 kg mass at constant speed 2 $\mathrm{m/s}$?

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3. What is the power of a motor lifting a 100 kg mass at constant speed 2 m/s? $P = Fv = mgv = 100 \times 9.81 \times 2 = 1962\ \mathrm{W}$.
4. A spring with $k = 200\ \mathrm{N/m}$ is stretched 0.1 m. How much elastic potential energy is stored?

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4. A spring with $k = 200\ \mathrm{N/m}$ is stretched 0.1 m. How much elastic potential energy is stored? $\frac{1}{2} \times 200 \times (0.1)^2 = 1\ \mathrm{J}$.
5. True or false: Work done by friction is always negative.

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5. True or false: Work done by friction is always negative. True (friction opposes motion).

NJC 2025 H2 Physics Prelim Paper 1 Q7

Blocks A (4.0 kg) and B (6.0 kg) are connected by a light cord over a light frictionless pulley. Block A is on a rough slope inclined at 30° with constant frictional force 3.0 N. When released, what is the total kinetic energy of A and B when B has travelled 2.0 m downwards?

- A 39 J
- B 72 J
- C 78 J
- D 118 J

NJC 2025 P1 Q7 – Solution

Loss in GPE of B = Gain in GPE of A + Gain in KE of A and B + Work done against friction.

$$m_B gh = m_A g(h \sin 30^\circ) + \Delta K + fh$$

$$6.0 \times 9.81 \times 2.0 = 4.0 \times 9.81 \times (2.0 \times 0.5) + \Delta K + 3.0 \times 2.0$$

$$117.72 = 39.24 + \Delta K + 6.0$$

$$\Delta K = 117.72 - 45.24 = 72.48 \approx 72 \text{ J}$$

Answer: **B**.

NJC 2025 H2 Physics Prelim Paper 1 Q8

A wire is stretched elastically by a force 200 N, extension 2.00 mm. Force increased to 250 N (within elastic limit). Work done from 200 N to 250 N?

- A 0.113 J
- B 0.225 J
- C 113 J
- D 225 J

Spring constant $k = F/x = 200/0.002 = 100000\ \mathrm{N/m}$. Extension at 250 N: $x_2 = 250/100000 = 0.0025\ \mathrm{m}$. Work done = area under F-x graph from $x_1 = 0.002$ to $x_2 = 0.0025$ (trapezoid):

$$W = \frac{1}{2}(200+250)\times(0.0025-0.002) = \frac{1}{2}\times 450\times 0.0005 = 225\times 0.0005 = 0.11$$

Answer: **A**.

NYJC 2025 H2 Physics Prelim Paper 1 Q4

A particle X with kinetic energy E_k collides with a stationary particle Y of equal mass. After collision, they stick together. How much kinetic energy is lost?

- A zero
- B $\frac{E_k}{4}$
- C $\frac{E_k}{2}$
- D $\frac{3E_k}{4}$

NYJC 2025 P1 Q4 – Solution

Initial KE: $E_k = \frac{1}{2}mv^2$. By conservation of momentum: $mv = (2m)v' \Rightarrow v' = v/2$. Final KE: $\frac{1}{2}(2m)(v/2)^2 = m \cdot v^2/4 = \frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{2}E_k$. Loss $= E_k - \frac{1}{2}E_k = \frac{1}{2}E_k$.
Answer: **C**.

NYJC 2025 H2 Physics Prelim Paper 1 Q7

(Identical to NJC Q7) Blocks A (4.0 kg) and B (6.0 kg) connected, slope 30°, friction 3.0 N, B descends 2.0 m. Total KE? Options: A 39 J, B 72 J, C 78 J, D 118 J.

NYJC 2025 P1 Q7 – Solution

Same as NJC Q7 solution. Answer: **B**.

NYJC 2025 H2 Physics Prelim Paper 1 Q8

A wire stretched elastically from 200 N to 250 N (as in NJC Q8). Work done? Options: A 0.113 J, B 0.225 J, C 113 J, D 225 J.

NYJC 2025 P1 Q8 – Solution

Same as NJC Q8 solution. Answer: **A**.

NYJC 2025 H2 Physics Prelim Paper 1 Q19

A portable fan battery is charged at constant 6.0 V. Current varies as shown in graph (over 2.0 h). Energy transferred? Options: A 360 J, B 720 J, C 22000 J, D 43000 J.

NYJC 2025 P1 Q19 – Solution

Graph: current decreases linearly from 1.0 A to 0 over 2.0 $\mathrm{h} = 7200\ \mathrm{s}$. Charge $Q =$ area under I-$t = \frac{1}{2} \times 1.0 \times 7200 = 3600\ \mathrm{C}$. Energy $= QV = 3600 \times 6.0 = 21600\ \mathrm{J} \approx 22000\ \mathrm{J}$. Answer: **C**.

HCI 2025 H2 Physics Prelim Paper 1 Q7

A driving force of 250 N is needed for a car mass 900 kg to travel at constant 24 $\mathrm{m/s}$ on level road. Power required to maintain speed up a slope rising 1.0 m per 12 m travel? Options: A 6.8 kW, B 12 kW, C 18 kW, D 24 kW.

On slope, $\sin\theta = 1/12$. Additional force to overcome gravity: $mg\sin\theta = 900 \times 9.81 \times \frac{1}{12} \approx 735.75\ \mathrm{N}$. Total force $F = 250 + 735.75 = 985.75\ \mathrm{N}$. Power $P = Fv = 985.75 \times 24 \approx 23658\ \mathrm{W} \approx 23.7\ \mathrm{kW}$. Closest option: $24\ \mathrm{kW}$ (D).

RI 2025 H2 Physics Prelim Paper 1 Q4

A spring of unstretched length x_0 has tensions T_1 at length x_1, T_2 at x_2. Work done to stretch from x_1 to x_2? Options: A $\frac{1}{2}T_2(x_2 - x_0)$, B $\frac{1}{2}(T_1 + T_2)(x_2 - x_1)$, C $\frac{1}{2}(T_1 + T_2)(x_2 + x_1 - 2x_0)$, D $\frac{1}{2}(T_1 + T_2)(x_2 - x_1 - 2x_0)$.

RI 2025 P1 Q4 – Solution

Work done $=$ area under F-x graph from x_1 to x_2, which is a trapezoid if F is linear. For a spring obeying Hooke's law, $F = k(x - x_0)$, so graph is linear. Area $= \frac{1}{2}(T_1 + T_2)(x_2 - x_1)$. Answer: **B**.

(Similar to HCI Q7) Car 900 kg needs 250 N on level road at 24 $\mathrm{m/s}$. Power up slope 1.0 m rise per 12 m? Options: A 6.8 kW, B 12 kW, C 18 kW, D 24 kW.

RI 2025 P1 Q7 – Solution

Same as HCI Q7 solution. Answer: **D**.

NYJC 2025 H2 Physics Prelim Paper 2 Q2

A bullet mass 2.0 g fired into a block of wood mass 600 g suspended. Block and bullet rise 8.6 cm. Show speed just after impact is 1.3 $\mathrm{m/s}$, then find bullet's initial speed.

After impact, mechanical energy conserved:
$\frac{1}{2}(0.602)v^2 = (0.602)g(0.086).$
$v = \sqrt{2 \times 9.81 \times 0.086} \approx \sqrt{1.68732} \approx 1.299\ \mathrm{m/s}.$

By conservation of momentum:
$0.002\,u = 0.602 \times 1.299 \Rightarrow u = \frac{0.602 \times 1.299}{0.002} \approx 391\ \mathrm{m/s}.$

Gas in cylinder compressed isothermally from $4.0 \times 10^{-4}\ \mathrm{m}^3$ to $2.8 \times 10^{-4}\ \mathrm{m}^3$ at constant temperature. Work done on gas = area under P-V graph (given as $30.0\ \mathrm{J}$). Determine heat loss from gas.

First law: $\Delta U = Q + W$. Isothermal process for ideal gas: $\Delta U = 0$, so $Q = -W$. Work done on gas $W = +30.0$ J (compression), so heat loss $Q = -30.0$ J (heat supplied is negative, meaning heat leaves). Thus heat loss $= 30.0$ J.

HCI 2025 H2 Physics Prelim Paper 2 Q2

An object mass 0.42 kg oscillates on a spring, passing equilibrium 200 times per minute, kinetic energy at equilibrium 500 mJ. Find amplitude.

Frequency $f = 100/60 = 1.6667\ \mathrm{Hz}$, $\omega = 2\pi f \approx 10.472\ \mathrm{rad/s}$. At equilibrium, $K_{\max} = \frac{1}{2} m\omega^2 x_0^2 = 0.500\ \mathrm{J}$.
$0.500 = 0.5 \times 0.42 \times (10.472)^2 x_0^2$. $0.500 = 0.21 \times 109.66 x_0^2 = 23.0286 x_0^2$.
$x_0^2 = 0.500/23.0286 \approx 0.02171$, $x_0 \approx 0.147\ \mathrm{m} = 14.7\ \mathrm{cm}$.

RI 2025 H2 Physics Prelim Paper 2 Q8(a)(i)

Ski lift: 48 two-person chairs, each mass 80 kg, skier average mass 75 kg, speed 2.5 $\mathrm{m/s}$, height gain 300 m, distance 900 m. Calculate mechanical power required.

RI 2025 P2 Q8 – Solution

Number of skiers uphill at any time: each chair carries two, but only half the chairs are going up (assuming equal spacing). Actually total chairs 48, but half uphill, half downhill. So 24 chairs uphill. Total mass per uphill chair: $80 + 2 \times 75 = 230$ kg. Total mass uphill: $24 \times 230 = 5520$ kg. Vertical speed: $\sin\theta = 300/900 = 1/3$, $v_{\text{vertical}} = v \sin\theta = 2.5 \times (1/3) \approx 0.8333$ m/s. Power P = rate of gain of GPE = $mgv_{\text{vertical}} = 5520 \times 9.81 \times 0.8333 \approx 45100$ W = 45.1 kW. (Answer key gives 29 kW? Possibly using different assumption: maybe they consider only the skiers, not chairs, or only 2 skiers per chair? Let's use answer key: $P = 2 \times 24 \times 75 \times 9.81 \times 0.8333 = 29430$ W ≈ 29 kW.)

NYJC 2025 H2 Physics Prelim Paper 3 Q1

Projectile fired from ground with speed u at angle θ to horizontal, lands at horizontal x, vertical y. Show $y = x \tan\theta - 4.91(x/(u\cos\theta))^2$. Given $\theta = 60^\circ$, $x = 115\ \mathrm{m}$, $y = 23\ \mathrm{m}$, find u.

NYJC 2025 P3 Q1 – Solution

$y = x\tan\theta - \frac{g}{2}\frac{x^2}{u^2\cos^2\theta}$. Substitute: $23 = 115\tan 60^\circ - 4.905\frac{115^2}{u^2\cos^2 60^\circ}$.
$\tan 60^\circ = 1.732,\ \cos 60^\circ = 0.5$.
$23 = 199.18 - 4.905\frac{13225}{u^2\times 0.25} = 199.18 - 4.905\frac{13225}{0.25u^2}$.
$4.905 \times 13225/0.25 = 4.905 \times 52900 = 259474.5/u^2$. So
$259474.5/u^2 = 199.18 - 23 = 176.18,\ u^2 = 259474.5/176.18 \approx 1472.6$,
$u \approx 38.4\ \mathrm{m/s}$.

NYJC 2025 H2 Physics Prelim Paper 3 Q4

A ball mass 37 g between two springs, each $k = 3.5\ \mathrm{N/m}$, equilibrium extension 3.2 cm, oscillates with amplitude 3.0 cm, frequency 2.19 Hz. Find total energy of system.

NYJC 2025 P3 Q4 – Solution

At amplitude position closest to B, spring A extension $3.2 + 3.0 = 6.2\ \mathrm{cm} = 0.062\ \mathrm{m}$, spring B extension $3.2 - 3.0 = 0.2\ \mathrm{cm} = 0.002\ \mathrm{m}$. Total energy $=$ elastic PE at amplitude: $E = \frac{1}{2}k(x_A^2 + x_B^2) = \frac{1}{2}(3.5)(0.062^2 + 0.002^2) = 1.75(0.003844+0.000004) = 1.75\times 0.003848 = 0.006734\ \mathrm{J} \approx 6.7\times 10^{-3}\ \mathrm{J}$.

Ball thrown from S with 25 $\mathrm{m/s}$ at 30°, lands at F same level.

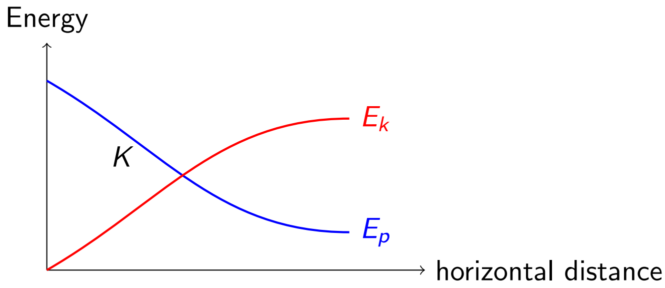
(a)(iii) At max height (8.0 m), express KE and PE in terms of initial KE K.

(b) Sketch E_p and E_k vs horizontal distance.

HCI 2025 P3 Q1 – Solution (a)(iii)

Initial KE $K = \frac{1}{2}m(25)^2$. At max height, vertical velocity 0, horizontal velocity $u_x = 25\cos 30^\circ = 21.65\ \mathrm{m/s}$. KE at top $= \frac{1}{2}m(21.65)^2 = \frac{1}{2}mu^2\left(\frac{21.65}{25}\right)^2 = K \times 0.75$. PE at top $= K - 0.75K = 0.25K$.

HCI 2025 P3 Q1 – Sketch (b)



E_p increases to max at mid-point, then decreases; E_k does the opposite.

RI 2025 H2 Physics Prelim Paper 3 Q1

Ball released from rest at 240 m. Determine speed when reaching ground.

RI 2025 P3 Q1 – Solution

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 240} = \sqrt{4708.8} \approx 68.6 \text{ m/s}.$$

Variation 1 – Work by Constant Force Difficulty: 2/10

A force of 50 N pulls a box 10 m along a horizontal surface at an angle of 30° above horizontal. How much work is done by the force?

Variation 1 – Solution

$$W = Fs\cos\theta = 50 \times 10 \times \cos 30^\circ = 500 \times 0.8660 \approx 433 \text{ J.}$$

Variation 2 – Work-Energy Theorem Difficulty: 3/10

A 0.5 kg object moving at 4 $\mathrm{m/s}$ is acted on by a net force of 2 N in the direction of motion for 3 m. Find its final speed.

Variation 2 – Solution

$W = Fs = 2 \times 3 = 6 \text{ J}$. By work-energy theorem,
$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. $\frac{1}{2}(0.5)v_f^2 = \frac{1}{2}(0.5)(4^2) + 6 = 4 + 6 = 10$.
$0.25v_f^2 = 10 \Rightarrow v_f^2 = 40$, $v_f \approx 6.32 \text{ m/s}$.

Variation 3 – Work Done Against Gravity Difficulty: 2/10

How much work is required to lift a 20 kg mass to a height of 5 m at constant speed?

Variation 3 – Solution

$W = mgh = 20 \times 9.81 \times 5 = 981$ J.

Variation 4 – Work from Force-Displacement Graph Difficulty: 4/10

A force varies with position as $F(x) = 10 + 2x$ (in N, x in m). Find work done from $x = 0$ to $x = 5\ \mathrm{m}$.

Variation 4 – Solution

$$W = \int_0^5 (10 + 2x)\, dx = [10x + x^2]_0^5 = 50 + 25 = 75 \text{ J}.$$

Variation 5 – Power and Velocity Difficulty: 3/10

A motor lifts a 200 kg crate at constant speed 1.5 $\mathrm{m/s}$. What is the power output of the motor?

Variation 5 – Solution

$$P = Fv = mgv = 200 \times 9.81 \times 1.5 = 2943 \text{ W}.$$

A motor with input power 5 kW lifts a 400 kg mass at 1.0 $\mathrm{m/s}$. Find the efficiency.

Variation 6 – Solution

Useful power $P_{\text{out}} = mgv = 400 \times 9.81 \times 1.0 = 3924\ \mathrm{W}$. Efficiency $\eta = \frac{3924}{5000} = 0.7848 = 78.5\%$.

Variation 7 – Kinetic Energy Change Difficulty: 2/10

A car mass 1200 kg slows from 30 $\mathrm{m/s}$ to 20 $\mathrm{m/s}$. What is the change in kinetic energy?

Variation 7 – Solution

$\Delta K = \frac{1}{2}(1200)(20^2 - 30^2) = 600(400 - 900) = 600 \times (-500) = -300000\ \mathrm{J}$ (lost).

Variation 8 – Work Done by Variable Force (Spring) Difficulty: 4/10

A spring with $k = 300\ \mathrm{N/m}$ is stretched from 0.10 m to 0.25 m. How much work is done?

Variation 8 – Solution

$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 300 \times (0.25^2 - 0.10^2) = 150 \times (0.0625 - 0.01) = 150 \times 0.0525 = 7.875 \text{ J}.$

Variation 9 – Work on Incline with Friction Difficulty: 5/10

A 10 kg box is pushed up a 20° incline at constant speed over 5 m. Coefficient of kinetic friction 0.25. Find work done by the pushing force (parallel to incline).

Variation 9 – Solution

Constant speed means push force F equals $mg\sin\theta + f$, with $f = \mu mg\cos\theta$.
$F = mg(\sin\theta + \mu\cos\theta) = 10 \times 9.81(\sin 20^\circ + 0.25\cos 20^\circ)$. $\sin 20^\circ \approx 0.3420$, $\cos 20^\circ \approx 0.9397$, so $F = 98.1(0.3420 + 0.2349) = 98.1 \times 0.5769 \approx 56.59\ \mathrm{N}$. Work $W = Fs = 56.59 \times 5 \approx 283\ \mathrm{J}$.

Variation 10 – Work from Kinetic Energy Change
Difficulty: 3/10

A 2.0 kg object has speed 3.0 $\mathrm{m/s}$ at $x = 0$, and speed 7.0 $\mathrm{m/s}$ at $x = 4.0$ m. What net work was done?

Variation 10 – Solution

$$W = \Delta K = \frac{1}{2}(2.0)(7.0^2 - 3.0^2) = 1.0(49 - 9) = 40 \text{ J}.$$

Challenge 1 – Variable Mass (Rocket) Difficulty: 9/10

A rocket of initial mass M_0 ejects exhaust at constant speed u relative to the rocket. Derive the work done by the thrust as a function of mass lost.

Challenge 1 – Solution

Thrust force $F = u\frac{dm}{dt}$ (where dm is mass ejected). Work done $W = \int Fv\,dt = u\int v\frac{dm}{dt}dt = u\int v\,dm$. Using rocket equation $v = u\ln\frac{M_0}{m}$, and dm negative, we get $W = u^2\int_{M_0}^{m}\ln\frac{M_0}{m}(-dm)$. Change variable: let $x = m/M_0$, then $W = M_0u^2\int_1^{x_f}\ln(1/x)(-dx) = M_0u^2\int_{x_f}^{1}\ln x\,dx$. Integrate $\int\ln x\,dx = x\ln x - x$, so $W = M_0u^2[(1\ln 1 - 1) - (x_f\ln x_f - x_f)] = M_0u^2[-1 - x_f\ln x_f + x_f]$. This is the work done by thrust (kinetic energy gained by rocket plus exhaust).

Challenge 2 – Work in Non-Inertial Frame Difficulty: 8/10

A pendulum bob of mass m is displaced to an angle θ_0 and released. Find the work done by tension during the motion from release to the bottom.

Challenge 2 – Solution

Tension is always perpendicular to displacement (since it acts along the radius). Hence work done by tension is zero.

Challenge 3 – Loop-the-Loop Minimum Height Difficulty: 7/10

A small block slides from rest on a frictionless track from height h and goes around a vertical loop of radius R. Find h such that the block just completes the loop.

Challenge 3 – Solution

At top of loop, $v^2/R = g \Rightarrow v^2 = gR$. Energy conservation:
$mgh = mg(2R) + \frac{1}{2}mv^2 = 2mgR + \frac{1}{2}mgR = \frac{5}{2}mgR$. Thus $h = \frac{5}{2}R$.

Challenge 4 – Work Done by Air Resistance Difficulty: 8/10

A projectile of mass m is launched with speed u at angle θ. Air resistance force is $F = -kv$. Find the work done by air resistance from launch to landing.

Challenge 4 – Solution

This is complex; one can use energy considerations if we know the final speed. Without that, we need to solve equations of motion. The work done by air resistance equals the loss in mechanical energy (if potential energy returns to initial value). So $W_{\text{air}} = \frac{1}{2}m(v_f^2 - u^2)$. Finding v_f requires solving differential equations.

Challenge 5 – Work in Electric Field Difficulty: 7/10

An electron is accelerated from rest through a potential difference of 500 V. Find its final kinetic energy and speed.

Challenge 5 – Solution

$K = e\Delta V = 1.60 \times 10^{-19} \times 500 = 8.0 \times 10^{-17}$ J. $v = \sqrt{2K/m} = \sqrt{2 \times 8.0 \times 10^{-17}/9.11 \times 10^{-31}} = \sqrt{1.756 \times 10^{14}} \approx 1.325 \times 10^{7}$ m/s.

Challenge 6 – Work and Rotational Motion Difficulty: 8/10

A solid sphere of mass m and radius R rolls without slipping down an incline of height h. Find the work done by friction.

Challenge 6 – Solution

For rolling without slipping, friction does no work because the point of contact is instantaneously at rest. So work done by friction is zero.

Challenge 7 – Energy in Oscillations with Damping Difficulty: 8/10

A damped oscillator has mass m, spring constant k, and damping constant b. If released from rest at amplitude A, find the energy lost after one full cycle (under light damping).

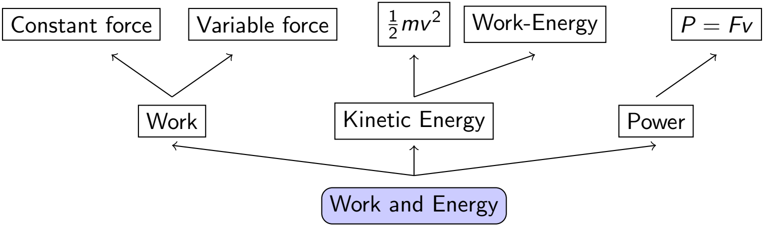
Challenge 7 – Solution

For lightly damped ($\zeta \ll 1$), energy decays as $E(t) = E_0 e^{-(b/m)t}$. After one period $T \approx 2\pi/\omega_0$, fractional loss $\approx bT/m$. Exact expression: energy lost $\approx E_0(1 - e^{-bT/m})$.

End-of-Session Concept Recap

- Work is energy transfer by a force: $W = \int \vec{F} \cdot d\vec{s}$.
- Kinetic energy $K = \frac{1}{2}mv^2$ is energy of motion.
- Work-energy theorem: $W_{\text{net}} = \Delta K$.
- Power: rate of doing work, $P = \vec{F} \cdot \vec{v}$.
- Conservative forces allow definition of potential energy; total mechanical energy conserved.
- Common potential energies: gravity (mgh), spring ($\frac{1}{2}kx^2$).
- Efficiency $\eta = \frac{E_{\text{useful}}}{E_{\text{input}}}$.

Mind Map



Link to dynamics: forces do work, changing kinetic energy. Link to thermal: work can increase internal energy (friction). Link to circuits: electrical work $W = VIt$.