H2 Physics
Kinematics

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Brain Dump

What do you know about kinematics?

Take a few minutes to jot down everything you remember:

- Definitions (displacement, velocity, acceleration)
- Equations of motion for constant acceleration
- Graphs (s–t, v–t, a–t)
- Projectile motion (independence of horizontal and vertical components)
- Free fall under gravity
- Relative motion
- Air resistance and terminal velocity

Math Checklist

Before diving into physics, ensure you are comfortable with:

- Solving linear and quadratic equations
- Trigonometry: $\sin\theta$, $\cos\theta$, $\tan\theta$
- Small angle approximations
- Vector addition and resolution
- Differentiation: $\frac{d}{dt}$ as rate of change
- Integration: area under v–t gives displacement
- Logarithms and exponentials (for variable acceleration)
- Significant figures and uncertainties
- Interpreting graphs: gradient, area under curve

Building Intuition

Real-world physics applications:

- Free fall: Dropping a ball from a height – how long does it take to hit the ground? What if thrown upward?
- Projectile motion: A basketball shot, a javelin throw, or a cannonball – the path is parabolic (ignoring air resistance).
- Terminal velocity: Sky divers reach a constant speed when air resistance balances weight.
- Relative motion: Overtaking a car on a highway – the relative speed determines the time to pass.

Why does a projectile follow a parabola?
Because horizontal velocity is constant while vertical acceleration is constant.

Formalization – Definitions

Displacement s

Vector quantity representing the change in position. Unit: m.

Velocity v

Rate of change of displacement: $v = \frac{ds}{dt}$. Unit: $\mathrm{m\,s^{-1}}$.

Acceleration a

Rate of change of velocity: $a = \frac{dv}{dt}$. Unit: $\mathrm{m\,s^{-2}}$.

Equations for uniform acceleration (a constant)

$$\begin{aligned} v &= u + at \\ s &= ut + \tfrac{1}{2}at^2 \\ v^2 &= u^2 + 2as \\ s &= \tfrac{1}{2}(u+v)t \end{aligned}$$

Formalization – Projectile Motion

- Resolve initial velocity into components: $u_x = u\cos\theta$, $u_y = u\sin\theta$.
- Horizontal motion: $a_x = 0$, $v_x = u_x$, $s_x = u_x t$.
- Vertical motion: $a_y = -g$, $v_y = u_y - gt$, $s_y = u_y t - \frac{1}{2}gt^2$.
- Time of flight, maximum height, range can be derived.

Example

A ball thrown at $20\,\mathrm{m\,s^{-1}}$ at 30° to horizontal. Find maximum height.

Micro-Testing – Quick Checks

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3. The gradient of a displacement–time graph gives velocity.
4. In projectile motion, the vertical component of velocity at the highest point is zero.
5. Two cars move toward each other at $20\,\mathrm{m\,s^{-1}}$ each. Their relative speed is $40\,\mathrm{m\,s^{-1}}$.

NJC 2025 H2 Physics Prelim Paper 1 Q3

A student throws a stone upwards at an initial speed of $15.0\,\mathrm{m\,s^{-1}}$. What is the displacement of the stone after $2.00\,\mathrm{s}$?

- A 1.12 m
- B 10.4 m
- C 11.5 m
- D 12.6 m

NJC 2025 P1 Q3 – Solution

Given: $u = 15.0\,\mathrm{m\,s^{-1}}$, $t = 2.00\,\mathrm{s}$, $a = -g = -9.81\,\mathrm{m\,s^{-2}}$.

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
&= (15.0)(2.00) + \frac{1}{2}(-9.81)(2.00)^2 \\
&= 30.0 - 19.62 = 10.38\,\mathrm{m} \approx 10.4\,\mathrm{m}
\end{aligned}
$$

Answer: **B**.

A stone falls vertically and strikes soft ground with speed u. The stone experiences constant deceleration until it comes to rest. Which graph shows the variation of speed v with distance s below the ground surface?
(Note: The original question includes four graphs. We reproduce the correct one conceptually.)

NYJC 2025 P1 Q3 – Solution

Take downward as positive. Below ground, acceleration is upward (negative). Let acceleration be $-\alpha$.

$$v^2 = u^2 + 2(-\alpha)s = u^2 - 2\alpha s$$

$$v = \sqrt{u^2 - 2\alpha s}$$

This is a decreasing square-root function, starting at $v = u$ when $s = 0$ and ending at $v = 0$ when $s = \frac{u^2}{2\alpha}$. The correct graph is option **A** (as per answer key).

A car X travels at constant speed u along a straight road. At time $t = 0$, a second car Y is a distance d_0 behind car X and travels at speed v ($v < u$). Car Y begins to accelerate with constant acceleration and overtakes car X at $t = T$. Which graph best shows the variation with time t of the distance d between the cars?
(Again, four graph options; we describe the correct shape.)

HCI 2025 P1 Q2 – Solution

Initially, distance $d = d_0$. Since $v < u$, d initially increases (Y slower). As Y accelerates, its speed eventually exceeds u, so d starts decreasing and becomes zero at overtaking. The correct graph shows d first rising with decreasing slope, then falling with increasing slope (concave up then concave down). Option **A** matches (as per answer key).

Two planets X and Y travel anticlockwise in circular orbits about a star. Radii ratio 3:1. At $t = 0$ they are aligned with the star. At $t = t_1$, X has moved 90°. Which diagram shows position of Y at $t = t_1$?
(We have the answer from the solution: Y moves through an angle such that it is 108° ahead? Actually we need to present the solution.)

RI 2025 P1 Q2 – Solution

Using Kepler's third law: $T^2 \propto r^3$. So $T_Y/T_X = (r_Y/r_X)^{3/2} = (1/3)^{3/2} = 1/\sqrt{27} \approx 0.192$. In the same time t_1, X covers 90°, so Y covers $90^\circ/0.192 \approx 468^\circ$, i.e., $468^\circ - 360^\circ = 108^\circ$ beyond its starting point. Hence Y is 108° ahead of its initial position. The correct diagram shows Y at an angle of 108° from the initial line. (Answer key: **B** for the specific diagram.)

NJC 2025 H2 Physics Prelim Paper 2 Q1

A small pellet of mass 8.00×10^{-3} kg is projected at angle θ above horizontal with speed u. It reaches a maximum height of 10.0 m and speed $5.00\,\mathrm{m\,s^{-1}}$ at maximum height. Air resistance negligible.

(a)(i) Show $u = 14.9\,\mathrm{m\,s^{-1}}$.

(a)(ii) Calculate θ.

(a)(iii) Time from launch to impact.

(a)(iv) Average rate of change of momentum.

(b) Complete displacement–time graph with air resistance.

NJC 2025 P2 Q1 – Solution (i)

At maximum height, vertical velocity $v_y = 0$, horizontal velocity $v_x = u\cos\theta = 5.00\,\mathrm{m\,s^{-1}}$. Using energy conservation: gain in GPE = loss in KE.

$$mg(10.0) = \frac{1}{2}mu^2 - \frac{1}{2}m(5.00)^2$$

Cancel m:

$$9.81 \times 10.0 = \frac{1}{2}u^2 - \frac{1}{2}(25.0)$$

$$98.1 = 0.5u^2 - 12.5$$

$$0.5u^2 = 110.6 \Rightarrow u^2 = 221.2 \Rightarrow u \approx 14.87\,\mathrm{m\,s^{-1}} \approx 14.9\,\mathrm{m\,s^{-1}}$$

NJC 2025 P2 Q1 – Solution (ii)

From $v_x = u\cos\theta = 5.00$, we have $\cos\theta = \frac{5.00}{14.873} \approx 0.3362$, so $\theta \approx \cos^{-1}(0.3362) \approx 70.4^\circ$. Alternatively, using vertical motion: $u_y = u\sin\theta$, at max height $v_y = 0$, $s_y = 10.0$, $a = -9.81$:

$$0 = u_y^2 + 2(-9.81)(10.0) \Rightarrow u_y = \sqrt{196.2} \approx 14.005$$

Then $\sin\theta = u_y/u = 14.005/14.873 \approx 0.9416$, $\theta \approx 70.3^\circ$ (rounding).

NJC 2025 P2 Q1 – Solution (iii)

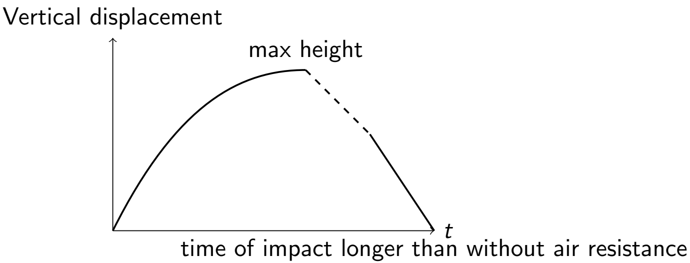
Time to return to launch height: use $s_y = u_y t + \frac{1}{2}at^2$ with $s_y = 0$, $u_y = u \sin\theta = 14.005$, $a = -9.81$.

$$0 = 14.005t - 4.905t^2 \Rightarrow t(14.005 - 4.905t) = 0$$

$t = 0$ or $t = 14.005/4.905 \approx 2.855\,\mathrm{s}$. Time of flight is $2.86\,\mathrm{s}$.

NJC 2025 P2 Q1 – Solution (iv)

Average rate of change of momentum = resultant force = weight = $mg = 8.00 \times 10^{-3} \times 9.81 = 0.0785\,\text{N}$.



Air resistance causes lower maximum height and longer total time of flight.

NYJC 2025 H2 Physics Prelim Paper 3 Q1

A projectile is fired from ground level with initial velocity u at angle θ to horizontal. It strikes a target at horizontal displacement x and vertical height y. Neglecting air resistance, show that

$$y = x\tan\theta - 4.91\left(\frac{x}{u\cos\theta}\right)^2$$

Given $\theta = 60^\circ$, $x = 115\,\text{m}$, $y = 23\,\text{m}$, calculate u.
Then sketch v_y vs t with and without air resistance.

NYJC 2025 P3 Q1 – Solution

From $x = u\cos\theta \cdot t$, we get $t = \frac{x}{u\cos\theta}$.
Vertical motion: $y = u\sin\theta \cdot t - \frac{1}{2}gt^2$. Substitute t:

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2 = x\tan\theta - \frac{g}{2}\frac{x^2}{u^2\cos^2\theta}$$

With $g = 9.81$, $\frac{g}{2} = 4.905 \approx 4.91$.
Given $\theta = 60^\circ$, $x = 115$, $y = 23$:

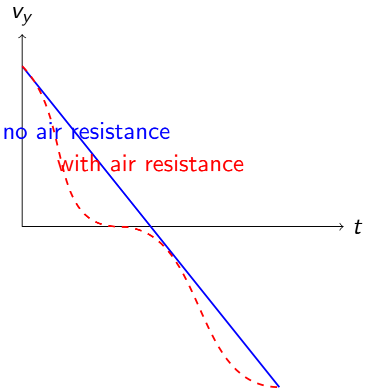
$$23 = 115\tan 60^\circ - 4.91\left(\frac{115}{u\cos 60^\circ}\right)^2$$

$$23 = 115 \times 1.732 - 4.91\left(\frac{115}{0.5u}\right)^2 = 199.18 - 4.91\left(\frac{230}{u}\right)^2$$

$$4.91\left(\frac{230}{u}\right)^2 = 199.18 - 23 = 176.18$$

$$\left(\frac{230}{u}\right)^2 = 35.88 \Rightarrow \frac{230}{u} = 5.990 \Rightarrow u \approx 38.4\,\mathrm{m\,s^{-1}}$$

NYJC 2025 P3 Q1 – v_y vs t sketch



With air resistance, v_y decreases faster, peak occurs earlier, and final speed smaller.

A ball is thrown from point S with initial velocity $25\,\text{m}\,\text{s}^{-1}$ at 30.0° to horizontal. Points S and F are at same level.

(a)(i) Calculate vertical component of initial velocity.

(a)(ii) Show maximum height is 8.0 m.

(a)(iii) At height 8.0 m, express KE and PE in terms of initial KE K.

(b) Sketch E_p and E_k vs horizontal distance.

HCI 2025 P3 Q1 – Solution (i)(ii)

(i) $u_y = u \sin 30^\circ = 25 \times 0.5 = 12.5\,\mathrm{m\,s^{-1}}$.
(ii) At max height, $v_y = 0$: $0 = u_y^2 - 2gs_y$
$s_y = \frac{u_y^2}{2g} = \frac{12.5^2}{2\times 9.81} = \frac{156.25}{19.62} \approx 7.96 \approx 8.0\,\mathrm{m}$.

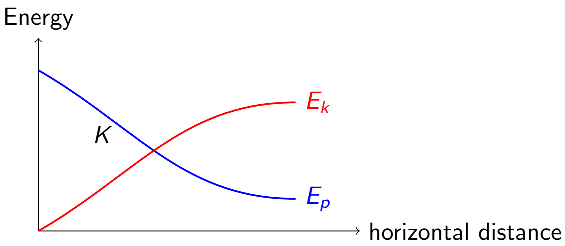
HCI 2025 P3 Q1 – Solution (iii)

Initial KE $K = \frac{1}{2}mu^2$. At max height, speed is horizontal component $u_x = u\cos 30^\circ = 25 \times 0.866 = 21.65$ m/s.

$$KE_{\text{top}} = \frac{1}{2}m(21.65)^2 = \frac{1}{2}mu^2\left(\frac{21.65}{25}\right)^2 = K \times 0.75$$

$$PE_{\text{top}} = K - KE_{\text{top}} = 0.25K$$

HCI 2025 P3 Q1 – Sketch (b)



E_p increases to max at mid-point, then decreases; E_k does the opposite.

HCI 2025 H2 Physics Prelim Paper 3 Q2(b)

Cars A and B approach a junction. Car A travels east at $40.0\,\mathrm{km\,h^{-1}}$, car B travels northwest at $50.0\,\mathrm{km\,h^{-1}}$. Determine the velocity of car A relative to car B.

HCI 2025 P3 Q2(b) – Solution

Let east be $+x$, north be $+y$.

$$\vec{v}_A = (40.0, 0) \text{ km/h}$$

$$\vec{v}_B = (50.0\cos 135^\circ, 50.0\sin 135^\circ) = (-35.355, 35.355) \text{ km/h}$$

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = (40.0 - (-35.355), 0 - 35.355) = (75.355, -35.355)$$

Magnitude: $\sqrt{75.355^2 + 35.355^2} \approx 83.2\,\text{km}\,\text{h}^{-1}$. Direction: $\theta = \tan^{-1}\left(\frac{-35.355}{75.355}\right) \approx -25.1^\circ$ (i.e., 25.1° south of east, or bearing 115.1°).

RI 2025 H2 Physics Prelim Paper 3 Q1

A ball is released from rest at the 80th floor (height 240 m). Air resistance negligible.

(a)(i) Time taken to fall from 60th to 50th floor (each floor 3.0 m).

(a)(ii) Explain why time from 50th to 40th floor is shorter.

(a)(iii) Speed when reaching ground.

(b) Sketch displacement–time graph with air resistance.

RI 2025 P3 Q1 – Solution (i)

Height of 60th floor: $60 \times 3 = 180\,\text{m}$ from ground.
Height of 50th floor: $50 \times 3 = 150\,\text{m}$ from ground.
Time to fall to 60th floor: $180 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2\times180}{9.81}} \approx 6.06$ s.
Time to fall to 50th floor: $150 = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \sqrt{\frac{300}{9.81}} \approx 5.53$ s.
Time between floors: $t_1 - t_2 = 0.53$ s.

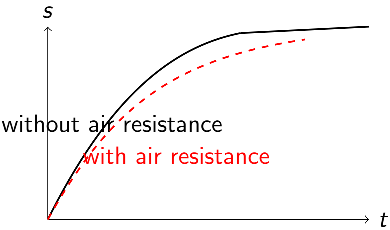
RI 2025 P3 Q1 – Solution (ii)

As the ball falls, it accelerates, so its speed increases. Therefore, it covers the same distance (10 m) in less time when it is lower.

RI 2025 P3 Q1 – Solution (iii)

Speed at ground: $v = \sqrt{2g(240)} = \sqrt{2 \times 9.81 \times 240} \approx \sqrt{4708.8} \approx 68.6\,\mathrm{m\,s^{-1}}$.

RI 2025 P3 Q1 – Sketch (b)



Air resistance reduces acceleration, so graph is less steep and final displacement reached later.

RI 2025 H2 Physics Prelim Paper 3 Q3(c)

An archer fires an arrow at $52\,\mathrm{m\,s^{-1}}$ at 15° above horizontal from height $1.5\,\mathrm{m}$. The arrow hits a tree at height $8.0\,\mathrm{m}$. Mass $32\,\mathrm{g}$. Neglect air resistance.

(i) Determine kinetic energy just before hitting tree.

(ii) Calculate distance of tree from archer.

RI 2025 P3 Q3(c) – Solution (i)

Use energy conservation between launch and impact. Loss in KE = gain in GPE:

$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = mg(h_f - h_i)$$

$$\frac{1}{2}(0.032)v_f^2 = \frac{1}{2}(0.032)(52)^2 - (0.032)(9.81)(8.0 - 1.5)$$

$$0.016v_f^2 = 0.016 \times 2704 - 0.032 \times 9.81 \times 6.5$$

$$0.016v_f^2 = 43.264 - 2.04048 = 41.22352$$

$$v_f^2 = 2576.47 \Rightarrow v_f \approx 50.76 \text{ m/s}$$

KE at tree = $\frac{1}{2}(0.032)(50.76)^2 \approx 41.2$ J.

RI 2025 P3 Q3(c) – Solution (ii)

Vertical motion: $s_y = u_y t - \frac{1}{2}gt^2$ with $s_y = 8.0 - 1.5 = 6.5$ m, $u_y = 52\sin 15^\circ \approx 13.46$ m/s.

$$6.5 = 13.46t - 4.905t^2$$

$$4.905t^2 - 13.46t + 6.5 = 0$$

Solve: $t = \frac{13.46 \pm \sqrt{13.46^2 - 4\times 4.905\times 6.5}}{2\times 4.905} = \frac{13.46 \pm \sqrt{181.17 - 127.53}}{9.81} = \frac{13.46 \pm \sqrt{53.64}}{9.81}$

$\sqrt{53.64} \approx 7.324$, so $t_1 = (13.46 - 7.324)/9.81 \approx 0.626$ s,

$t_2 = (13.46 + 7.324)/9.81 \approx 2.12$ s (later time corresponds to arrow coming down). Use $t = 0.626$ s. Horizontal distance:

$x = u_x t = 52\cos 15^\circ \times 0.626 \approx 50.24 \times 0.626 \approx 31.4\,\text{m}$.

Free Fall – Variation 1 Difficulty: 3/10

A ball is thrown vertically upward with initial speed $18.0\,\mathrm{m\,s^{-1}}$. Determine:

1. The maximum height reached.
2. The time taken to return to the thrower's hand.
3. The velocity after $2.5\,\mathrm{s}$.

Free Fall – Variation 1 – Solution

Take upward positive, $g = 9.81\ \text{m/s}^2$.

1. At max height $v = 0$: $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = \frac{18^2}{2\times 9.81} \approx \frac{324}{19.62} \approx 16.5\,\text{m}$.
2. Total time of flight: $s = 0$ when returns, $0 = ut - \frac{1}{2}gt^2 \Rightarrow t(u - \frac{1}{2}gt) = 0$, $t = 0$ or $t = 2u/g = 36/9.81 \approx 3.67\,\text{s}$.
3. $v = u - gt = 18 - 9.81 \times 2.5 = 18 - 24.525 = -6.53\,\text{m}\,\text{s}^{-1}$ (downward).

Free Fall – Variation 2 Difficulty: 4/10

A car moving at $30\,\text{m}\,\text{s}^{-1}$ applies brakes and decelerates uniformly at $4.0\,\text{m}\,\text{s}^{-2}$ until it stops. Sketch the graph of speed v versus distance s travelled after braking. Write the equation relating v and s.

Free Fall – Variation 2 – Solution

Using $v^2 = u^2 + 2as$ with $a = -4.0$ m/s^2, $u = 30$ m/s:

$$v^2 = 900 - 8.0s$$

So $v = \sqrt{900 - 8.0s}$ for $0 \leq s \leq 900/8 = 112.5$ m. The graph is a decreasing square-root function.

Relative Motion – Variation 3 Difficulty: 4/10

A police car travelling at $80\,\text{km}\,\text{h}^{-1}$ is passed by a speeding car at $140\,\text{km}\,\text{h}^{-1}$. At that instant, the police car begins accelerating uniformly at $2.0\,\text{m}\,\text{s}^{-2}$. How long does it take the police car to catch the speeding car?

Relative Motion – Variation 3 – Solution

Convert speeds to m/s: 80 km/h = 22.22 m/s, 140 km/h = 38.89 m/s. Let t be time after passing. Speeding car position: $s_s = 38.89t$. Police car position: $s_p = 22.22t + \frac{1}{2}(2.0)t^2 = 22.22t + t^2$. Set $s_p = s_s$:
$22.22t + t^2 = 38.89t \Rightarrow t^2 - 16.67t = 0 \Rightarrow t(t - 16.67) = 0$, so $t = 16.67$ s.

Projectile Motion – Variation 4 Difficulty: 5/10

A ball is kicked from ground level at $20\,\mathrm{m\,s^{-1}}$ at 40° above horizontal. Determine:

1. The time of flight.
2. The range.
3. The speed at impact.
4. The angle of the velocity vector just before hitting the ground.

Projectile Motion – Variation 4 – Solution

$u_x = 20\cos 40^\circ \approx 15.32$ m/s, $u_y = 20\sin 40^\circ \approx 12.86$ m/s.

1. Time of flight: $T = \frac{2u_y}{g} = \frac{2\times 12.86}{9.81} \approx 2.62\,\text{s}$.
2. Range: $R = u_x T = 15.32 \times 2.62 \approx 40.1\,\text{m}$.
3. Speed at impact: horizontal unchanged, vertical $v_y = -u_y = -12.86$ m/s, so $v = \sqrt{15.32^2 + 12.86^2} = \sqrt{234.7 + 165.4} = \sqrt{400.1} \approx 20.0$ m/s (by energy conservation, it's the same as initial speed).
4. Angle below horizontal: $\tan\theta = \frac{12.86}{15.32} \approx 0.839$, $\theta \approx 40.0^\circ$ below horizontal.

Relative Motion – Variation 5 Difficulty: 4/10

A plane flies due north at $200\,\mathrm{km\,h^{-1}}$ relative to the air. There is a wind blowing from the west at $50\,\mathrm{km\,h^{-1}}$. What is the velocity (magnitude and direction) of the plane relative to the ground?

Relative Motion – Variation 5 – Solution

Velocity of plane relative to air: $\vec{v}_{p/a} = (0, 200)$ km/h. Wind velocity (air relative to ground): $\vec{v}_{a/g} = (50, 0)$ (eastward). Then $\vec{v}_{p/g} = \vec{v}_{p/a} + \vec{v}_{a/g} = (50, 200)$. Magnitude: $\sqrt{50^2 + 200^2} = \sqrt{2500 + 40000} = \sqrt{42500} \approx 206.2$ km/h. Direction: $\theta = \tan^{-1}(50/200) \approx 14.0^\circ$ east of north.

A stone is dropped from the top of a building. It passes a window of height 2.0 m in 0.15 s. How far below the top of the building is the top of the window?

Free Fall – Variation 6 – Solution

Let h be distance from top to top of window. At top of window, velocity $v_1 = \sqrt{2gh}$. The stone takes $t = 0.15$ s to fall the window height 2.0 m. Using $s = v_1 t + \frac{1}{2}gt^2$:

$$2.0 = \sqrt{2gh} \times 0.15 + \frac{1}{2}(9.81)(0.15)^2$$

$$2.0 = 0.15\sqrt{19.62h} + 0.11036$$

$$1.88964 = 0.15\sqrt{19.62h}$$

$$\sqrt{19.62h} = 12.5976$$

$$19.62h = 158.7 \Rightarrow h \approx 8.09\,\text{m}$$

Projectile Motion – Variation 7 Difficulty: 6/10

A ball is thrown from a height of 2.0 m above ground at $15\,\mathrm{m\,s^{-1}}$ and 50° above horizontal. It lands on a platform 3.5 m high. Find the horizontal distance to the platform.

Projectile Motion – Variation 7 – Solution

$u_x = 15\cos 50^\circ \approx 9.642$ m/s, $u_y = 15\sin 50^\circ \approx 11.49$ m/s. Vertical displacement from launch to landing: $\Delta y = 3.5 - 2.0 = 1.5$ m (upward). Use $y = u_y t - \frac{1}{2}gt^2$:

$$1.5 = 11.49t - 4.905t^2$$

$$4.905t^2 - 11.49t + 1.5 = 0$$

Discriminant: $11.49^2 - 4 \times 4.905 \times 1.5 = 132.0 - 29.43 = 102.57$, $\sqrt{102.57} \approx 10.13$. $t = \frac{11.49 \pm 10.13}{9.81}$. Two positive roots: $t_1 = 0.139$ s, $t_2 = 2.20$ s. The shorter time is on the way up, longer on way down. Use $t = 2.20$ s (landing after peak). Horizontal distance: $x = 9.642 \times 2.20 \approx 21.2\,\text{m}$.

Projectile on Incline – Challenge 1 Difficulty: 8/10

A particle is projected from point O on an inclined plane of angle α with speed u at an angle θ to the horizontal. It lands on the same inclined plane. Derive an expression for the range R along the incline.

Projectile on Incline – Challenge 1 – Solution

Set up coordinates: x along incline up, y perpendicular to incline. Gravity components: $g_x = -g\sin\alpha$, $g_y = -g\cos\alpha$. Initial velocity components: $u_x = u\cos(\theta-\alpha)$, $u_y = u\sin(\theta-\alpha)$. Equations: $x = u_x t + \frac{1}{2}g_x t^2$, $y = u_y t + \frac{1}{2}g_y t^2$. When particle lands on incline, $y = 0$ (since it started on incline). Solve $0 = u_y t + \frac{1}{2}g_y t^2$ gives $t = 0$ or $t = -\frac{2u_y}{g_y} = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$. Then

$R = x(t) = u\cos(\theta-\alpha)\cdot\frac{2u\sin(\theta-\alpha)}{g\cos\alpha} + \frac{1}{2}(-g\sin\alpha)\left(\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}\right)^2$

$$R = \frac{2u^2}{g\cos\alpha}\left[\cos(\theta-\alpha)\sin(\theta-\alpha) - \sin\alpha\frac{\sin^2(\theta-\alpha)}{\cos\alpha}\right]$$

Simplify using $\cos(\theta-\alpha)\sin(\theta-\alpha) = \frac{1}{2}\sin 2(\theta-\alpha)$ and combine terms. Further manipulation yields $R = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$ (common form).

Non-constant Acceleration – Challenge 2 Difficulty: 9/10

A particle moves along the x-axis with acceleration $a = -kv^2$, where k is a positive constant. If it starts with speed u at $x = 0$, find the time t when its speed is $u/2$.

Non-constant Acceleration – Challenge 2 – Solution

$a = \frac{dv}{dt} = -kv^2$. Separate variables: $\frac{dv}{v^2} = -kdt$. Integrate: $\int_u^v v^{-2} dv = -k \int_0^t dt$ gives $\left[-v^{-1}\right]_u^v = -kt$. So $-\frac{1}{v} + \frac{1}{u} = -kt$, i.e., $\frac{1}{v} = \frac{1}{u} + kt$. When $v = u/2$, $\frac{2}{u} = \frac{1}{u} + kt \Rightarrow \frac{1}{u} = kt \Rightarrow t = \frac{1}{ku}$.

Two-dimensional Motion with Drag – Challenge 3 Difficulty: 9/10

A projectile of mass m is launched with speed u at angle θ. Air resistance force is proportional to velocity: $\vec{F} = -k\vec{v}$. Write the equations of motion and describe qualitatively how the trajectory differs from the vacuum case.

Two-dimensional Motion with Drag – Challenge 3 – Solution

Components: $m\ddot{x} = -k\dot{x}$, $m\ddot{y} = -mg - k\dot{y}$. These are linear ODEs. Solve: $\dot{x}(t) = u_x e^{-(k/m)t}$, so $x(t) = \frac{mu_x}{k}(1 - e^{-(k/m)t})$. $\dot{y}(t) = (u_y + \frac{mg}{k})e^{-(k/m)t} - \frac{mg}{k}$. Integrate to get $y(t)$. The trajectory is not symmetric; range is reduced, and the descent is steeper than ascent.

The velocity–time graph of a particle is a straight line from $(0, 10)$ to $(5, 0)$. Find the displacement and distance travelled.

Graphs – Variation 8 – Solution

The graph indicates constant deceleration. Displacement = area under graph = area of triangle $= \frac{1}{2} \times 5 \times 10 = 25$ m. Since velocity never changes sign, distance = displacement = 25 m.

A train accelerates uniformly from rest at $1.2\,\mathrm{m\,s^{-2}}$ for $20\,\mathrm{s}$, then travels at constant speed for $60\,\mathrm{s}$, then decelerates uniformly at $1.5\,\mathrm{m\,s^{-2}}$ until stop. Find total distance.

Multi-stage motion – Variation 9 – Solution

Stage 1: $v = at = 1.2 \times 20 = 24$ m/s, $s_1 = \frac{1}{2}at^2 = 0.5 \times 1.2 \times 400 = 240$ m. Stage 2: $s_2 = vt = 24 \times 60 = 1440$ m. Stage 3: deceleration $a = -1.5$ m/s^2, time to stop $t = 24/1.5 = 16$ s, $s_3 = \frac{1}{2}(v + 0)t = 0.5 \times 24 \times 16 = 192$ m. Total $= 240 + 1440 + 192 = 1872$ m.

A driver sees an obstacle and applies brakes after a reaction time of 0.50 s. The car decelerates at $5.0\,\mathrm{m\,s^{-2}}$ and stops in a total distance of 60 m. Find the initial speed.

Reaction time – Variation 10 – Solution

Let u be initial speed. During reaction: distance $s_1 = u \times 0.50$. During braking: $v^2 = u^2 + 2as_2$ with $v = 0$, $a = -5.0$, so $0 = u^2 - 10s_2 \Rightarrow s_2 = u^2/10$. Total $s_1 + s_2 = 0.5u + u^2/10 = 60$. Multiply by 10: $5u + u^2 = 600 \Rightarrow u^2 + 5u - 600 = 0$. Solve: $u = \frac{-5 \pm \sqrt{25+2400}}{2} = \frac{-5 \pm 49.24}{2}$, positive $u = 22.12$ m/s.

Relative Motion with Acceleration – Challenge 4 Difficulty: 8/10

Two cars start from rest from the same point. Car A accelerates at $2.0\,\mathrm{m\,s^{-2}}$ due east. Car B accelerates at $3.0\,\mathrm{m\,s^{-2}}$ due north. Find the velocity and acceleration of car B as observed from car A as functions of time.

Relative Motion with Acceleration – Challenge 4 – Solution

In ground frame: $\vec{v}_A = 2t\ \hat{i}$, $\vec{v}_B = 3t\ \hat{j}$. $\vec{a}_A = 2\hat{i}$, $\vec{a}_B = 3\hat{j}$. Relative velocity: $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = -2t\hat{i} + 3t\hat{j}$. Magnitude $= t\sqrt{4+9} = t\sqrt{13}$ m/s, direction $\theta = \tan^{-1}(3t/(-2t)) = \tan^{-1}(-1.5)$ but note quadrant (second quadrant) – constant direction. Relative acceleration: $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = -2\hat{i} + 3\hat{j}$, constant.

Projectile – Minimum Speed – Challenge 5 Difficulty: 8/10

A stunt driver wants to jump a gap of width L between two ramps of equal height. The ramps are inclined at θ to the horizontal. Show that the minimum speed required is $\sqrt{\frac{gL}{\sin 2\theta}}$.

Challenge 5 – Solution

The car leaves one ramp at speed u at angle θ, lands on other ramp at same height. Range formula: $R = \frac{u^2 \sin 2\theta}{g}$. For $R = L$, we get $u = \sqrt{\frac{gL}{\sin 2\theta}}$. This is the minimum because for any lower speed the range is less.

Variable Acceleration – Challenge 6 Difficulty: 9/10

A particle moves along a straight line with acceleration $a = 2t - 3$. At $t = 0$, $x = 0$ and $v = 4$ m/s. Find the time when the particle returns to the origin.

Challenge 6 – Solution

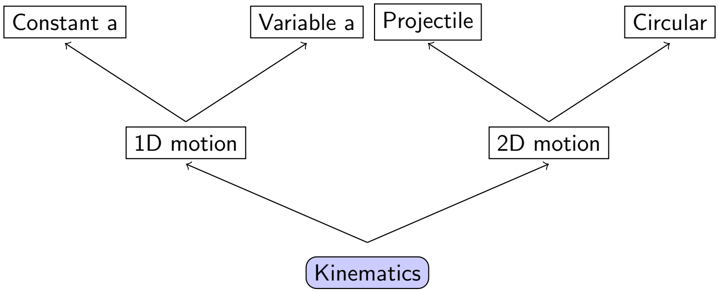
$a = dv/dt = 2t - 3$, integrate: $v = t^2 - 3t + C$. Using $v(0) = 4$, $C = 4$, so $v = t^2 - 3t + 4$. Then $x = \int v dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t + D$, $x(0) = 0 \Rightarrow D = 0$. Set $x = 0$: $\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t = 0 \Rightarrow t(\frac{1}{3}t^2 - \frac{3}{2}t + 4) = 0$. $t = 0$ is start. Solve quadratic: $\frac{1}{3}t^2 - 1.5t + 4 = 0 \Rightarrow t^2 - 4.5t + 12 = 0$, discriminant $20.25 - 48 = -27.75$ negative. So no real positive root; particle never returns to origin? Check: velocity always positive? $v = t^2 - 3t + 4$ has discriminant $9 - 16 = -7$, so $v > 0$ for all t, so particle always moves forward. So it never returns. That's an interesting outcome.

End-of-Session Concept Recap

Key takeaways:

- Kinematics describes motion without regard to causes.
- Equations of motion apply only when acceleration is constant.
- Graphs: slope of s–t gives v, slope of v–t gives a, area under v–t gives s.
- Projectile motion: horizontal and vertical components independent.
- Relative motion: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$.
- Air resistance leads to terminal velocity and asymmetric trajectories.

Mind Map



Linking to dynamics: forces cause acceleration (Newton's 2nd law).
Energy methods can also solve kinematic problems.