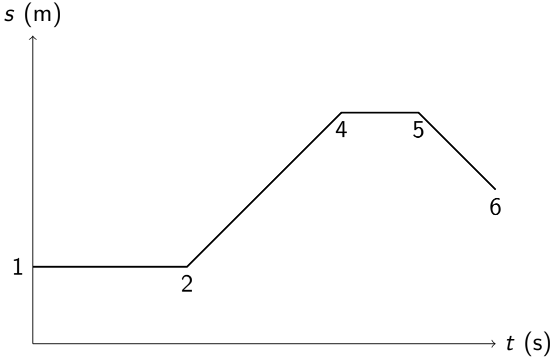
H2 Physics
Kinematics: Graphing

R.F.H.

March 16, 2026

Question 1

The displacement–time graph for a moving object is shown below.



Describe the motion during each time interval and determine the velocity in each segment.

Solution to Question 1

- 0–2 s: Displacement constant at 1 m $\rightarrow$ object stationary. Velocity $= 0$ m/s.
- 2–4 s: Displacement increases linearly from 1 m to 3 m $\rightarrow$ constant velocity.

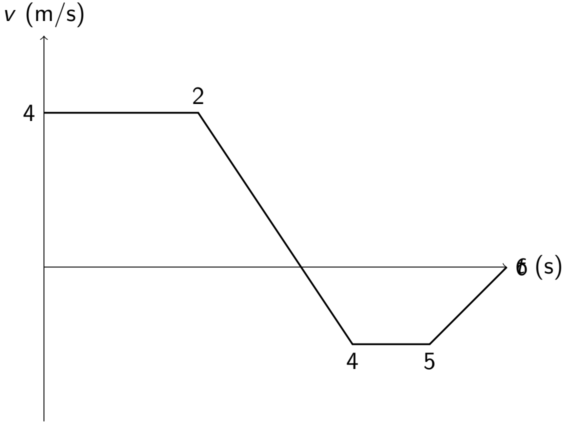
$$v = \frac{3-1}{4-2} = \frac{2}{2} = 1 \text{ m/s}$$

- 4–5 s: Displacement constant at 3 m $\rightarrow$ stationary. Velocity $= 0$ m/s.
- 5–6 s: Displacement decreases linearly from 3 m to 2 m $\rightarrow$ constant negative velocity.

$$v = \frac{2-3}{6-5} = \frac{-1}{1} = -1 \text{ m/s}$$

Question 2

The velocity–time graph of a particle is shown.



Find the acceleration in each phase and the total displacement.

Solution to Question 2

Acceleration = gradient of v–t graph.

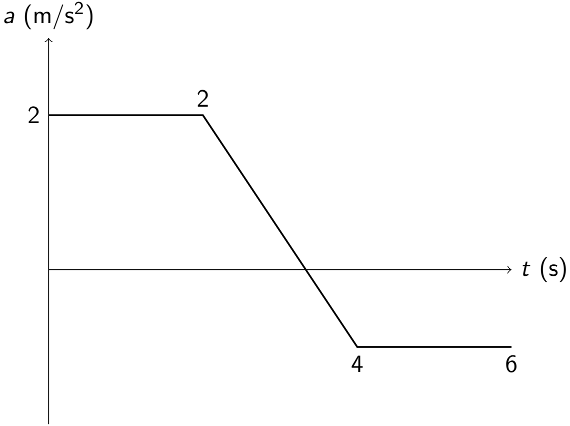
- 0–2 s: constant $v = 4$ m/s $\rightarrow a = 0$.
- 2–4 s: v changes from 4 to -1 m/s, $\Delta t = 2$ s $\rightarrow a = \frac{-1-4}{2} = -2.5$ m/s^2.
- 4–5 s: constant $v = -1$ m/s $\rightarrow a = 0$.
- 5–6 s: v changes from -1 to 0 m/s, $\Delta t = 1$ s $\rightarrow a = \frac{0-(-1)}{1} = 1$ m/s^2.

Displacement = area under v–t graph (positive above axis, negative below).

$$\begin{aligned}
&\text{0–2 s: } 4 \times 2 = 8 \text{ m} \\
&\text{2–4 s: } \frac{1}{2}(4 + (-1)) \times 2 = 3 \text{ m} \quad \text{(trapezoid)} \\
&\text{4–5 s: } (-1) \times 1 = -1 \text{ m} \\
&\text{5–6 s: } \frac{1}{2}(-1 + 0) \times 1 = -0.5 \text{ m} \\
&\text{Total } 8 + 3 - 1 - 0.5 = 9.5 \text{ m}
\end{aligned}$$

Question 3

The acceleration–time graph of a particle starting from rest is shown.



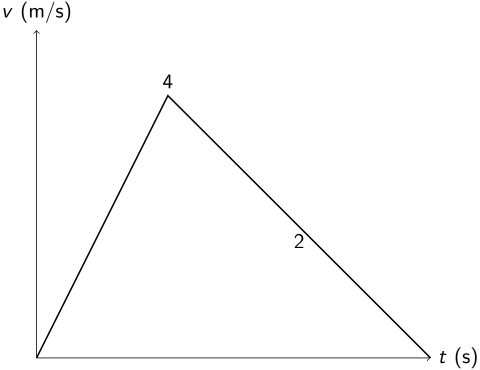
Sketch the velocity–time graph for $t = 0$ to $t = 6$ s.

Solution to Question 3

Velocity is the integral of acceleration. Starting from rest ($v = 0$ at $t = 0$).

- 0–2 s: constant $a = 2$ m/s^2 $\rightarrow$ v increases linearly: $v = 2t$. At $t = 2$, $v = 4$ m/s.
- 2–4 s: constant $a = -1$ m/s^2 $\rightarrow$ v decreases linearly from 4 m/s. At $t = 4$, $v = 4 + (-1) \times 2 = 2$ m/s.
- 4–6 s: constant $a = -1$ m/s^2 $\rightarrow$ v continues decreasing linearly. At $t = 6$, $v = 2 + (-1) \times 2 = 0$ m/s.

Sketch:



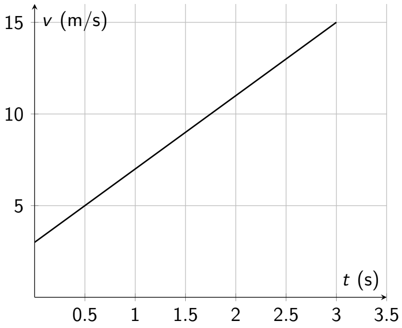
Question 4

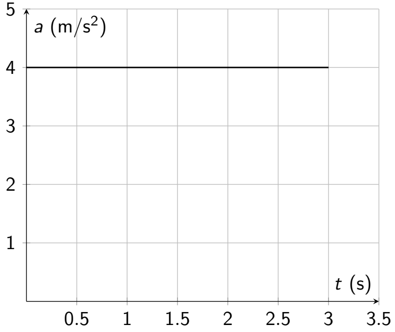
The displacement–time graph of an object moving in a straight line is given by $s = 2t^2 + 3t + 1$ (SI units).

1. Find the velocity at $t = 2$ s.
2. Find the acceleration.
3. Sketch the v–t and a–t graphs for $0 \leq t \leq 3$ s.

Solution to Question 4

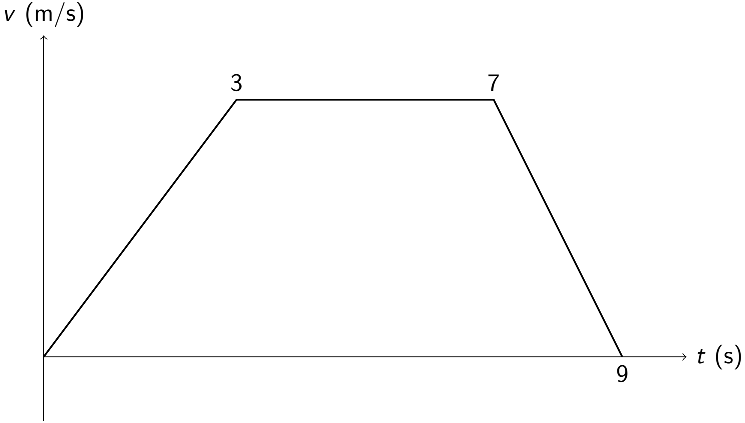
1. Velocity $v = \frac{ds}{dt} = 4t + 3$. At $t = 2$, $v = 4(2) + 3 = 11$ m/s.
2. Acceleration $a = \frac{dv}{dt} = 4$ m/s^2 (constant).
3. v–t: straight line from $(0, 3)$ to $(3, 15)$. a–t: horizontal line at $a = 4$.





Question 5

The velocity–time graph of a train is shown.



Describe the motion during each phase and calculate the total distance travelled.

Solution to Question 5

- 0–3 s: Constant acceleration from rest to 4 m/s. $a = (4 - 0)/3 = 1.33\ \text{m/s}^2$.
- 3–7 s: Constant velocity 4 m/s.
- 7–9 s: Constant deceleration from 4 m/s to 0. $a = (0 - 4)/2 = -2\ \text{m/s}^2$.

Distance = area under v–t graph:

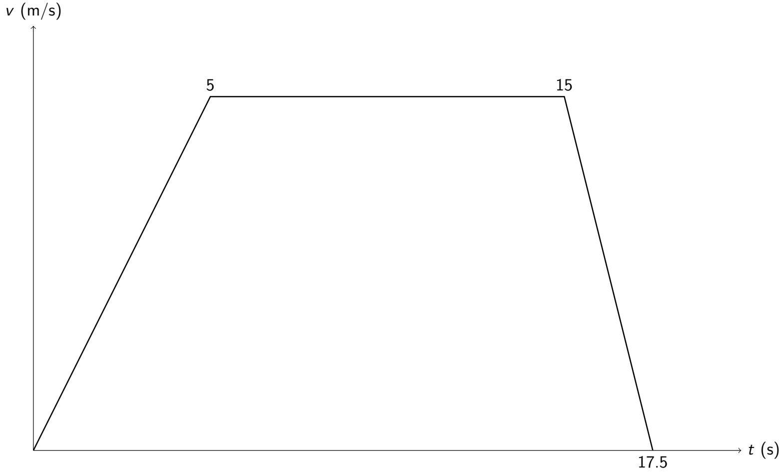
$$\begin{aligned} \text{0–3 s: triangle } & \frac{1}{2} \times 3 \times 4 = 6\ \text{m} \\ \text{3–7 s: rectangle } & 4 \times 4 = 16\ \text{m} \\ \text{7–9 s: triangle } & \frac{1}{2} \times 2 \times 4 = 4\ \text{m} \\ \text{Total } & 6 + 16 + 4 = 26\ \text{m} \end{aligned}$$

Question 6

A car starts from rest, accelerates uniformly at 2 m/s^2 for 5 s, then travels at constant speed for 10 s, and finally decelerates uniformly at 4 m/s^2 until it stops. Sketch the velocity–time graph and determine the total distance travelled.

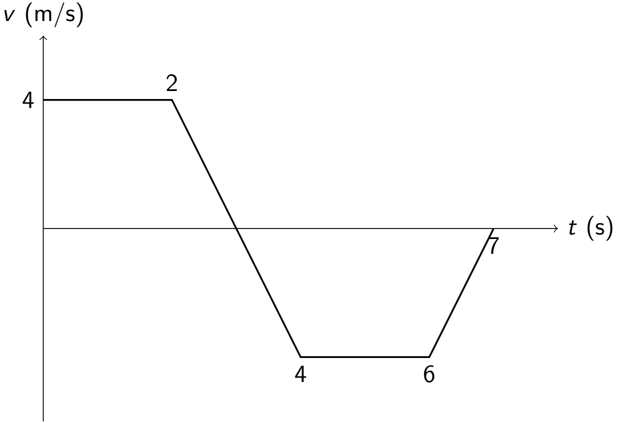
Solution to Question 6

Phase 1: $v = at = 2 \times 5 = 10$ m/s at $t = 5$ s. Distance $s_1 = \frac{1}{2} \times 5 \times 10 = 25$ m. Phase 2: constant $v = 10$ m/s for 10 s $\rightarrow s_2 = 10 \times 10 = 100$ m. Phase 3: deceleration $a = -4$ m/s^2 from 10 m/s to 0: time $t_3 = v/a = 10/4 = 2.5$ s. Distance $s_3 = \frac{1}{2} \times 2.5 \times 10 = 12.5$ m. Total distance $= 25 + 100 + 12.5 = 137.5$ m.



Question 7

The graph shows the velocity of a cyclist.



Find the total displacement and the total distance travelled.

Solution to Question 7

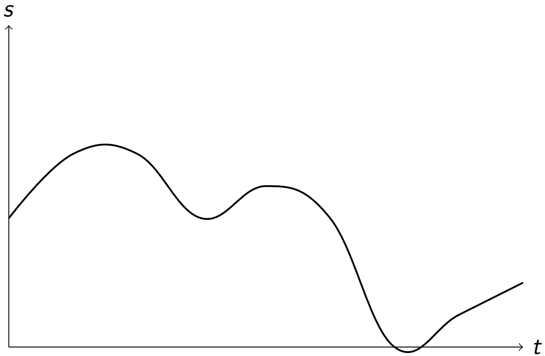
Displacement = area above axis minus area below axis.

- 0–2 s: rectangle $4 \times 2 = 8$ m.
- 2–4 s: line from $(2, 4)$ to $(4, -2)$: slope $= (-2-4)/(2) = -3$. Equation $v = 4 - 3(t-2)$. Area $= \int_2^4 (4-3(t-2))dt = \int_2^4 (10-3t)dt = [10t - \frac{3}{2}t^2]_2^4 = (40-24)-(20-6) = 16-14 = 2$ m.
- 4–6 s: constant $v = -2$ m/s, area $= -2 \times 2 = -4$ m.
- 6–7 s: triangle from -2 to 0, area $= \frac{1}{2} \times 1 \times (-2) = -1$ m.

Displacement $= 8 + 2 - 4 - 1 = 5$ m. Distance = sum of absolute areas: $8 + 2 + 4 + 1 = 15$ m.

Question 8

The displacement–time graph of a particle is shown. Indicate the intervals where velocity is positive, negative, or zero.



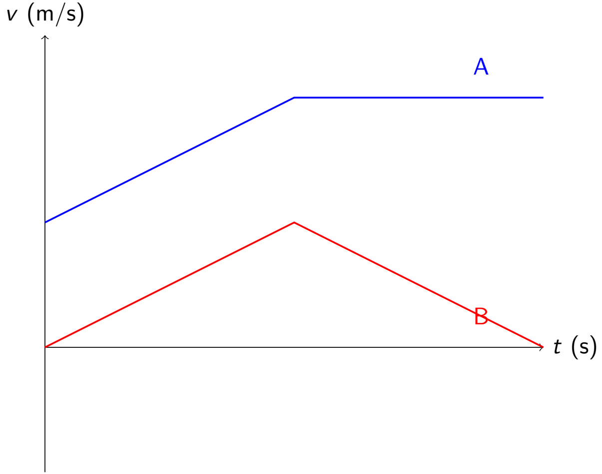
Solution to Question 8

Velocity is positive when s increases, negative when s decreases, zero when slope is horizontal.

- 0–1: increasing $\rightarrow$ positive
- 1–2: constant $\rightarrow$ zero
- 2–3: decreasing $\rightarrow$ negative
- 3–4: increasing $\rightarrow$ positive
- 4–5: decreasing $\rightarrow$ negative
- 5–6: decreasing steeply $\rightarrow$ negative
- 6–7: increasing $\rightarrow$ positive
- 7–8: increasing $\rightarrow$ positive

Question 9

Two cars A and B start from the same point. Their velocity–time graphs are shown.



Determine the time at which they have the same velocity and the time at which they meet again.

Solution to Question 9

- Velocity equations:

$$\text{A: } v_A = 2 + 0.5t \quad (0 \le t \le 4), \quad v_A = 4 \quad (t \ge 4)$$
$$\text{B: } v_B = 0.5t \quad (0 \le t \le 4), \quad v_B = 4 - 0.5t \quad (t \ge 4)$$

  Setting $v_A = v_B$:
  - For $t < 4$: $2 + 0.5t = 0.5t \Rightarrow 2 = 0$, impossible.
  - For $t \ge 4$: $4 = 4 - 0.5t \Rightarrow 0.5t = 0 \Rightarrow t = 0$, not valid.

  Hence they never have the same velocity.
- Displacement equations:

$$s_A = \begin{cases} 2t + 0.25t^2, & t \le 4 \\ 4t - 4, & t \ge 4 \end{cases}$$
$$s_B = \begin{cases} 0.25t^2, & t \le 4 \\ 4t - 0.25t^2 - 8, & t \ge 4 \end{cases}$$

  Setting $s_A = s_B$:
  - For $t \le 4$: $2t + 0.25t^2 = 0.25t^2 \Rightarrow 2t = 0 \Rightarrow t = 0$.
  - For $t \ge 4$: $4t - 4 = 4t - 0.25t^2 - 8 \Rightarrow -4 = -0.25t^2 - 8 \Rightarrow 0.25t^2 = -4$, no real solution.

  Thus they meet only at $t = 0$ (start).

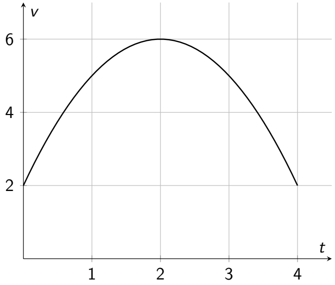
Question 10 (Challenging)

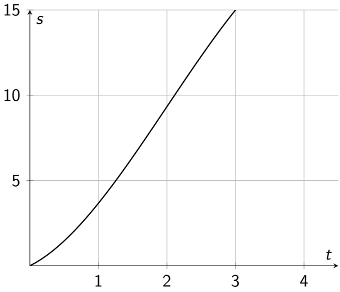
The acceleration of a particle moving along a straight line is given by $a = 4 - 2t$ (SI units) for $0 \leq t \leq 4$ s. At $t = 0$, $v = 2$ m/s and $s = 0$.

1. Find the velocity as a function of time.
2. Find the displacement as a function of time.
3. Sketch the v–t and s–t graphs.

Solution to Question 10

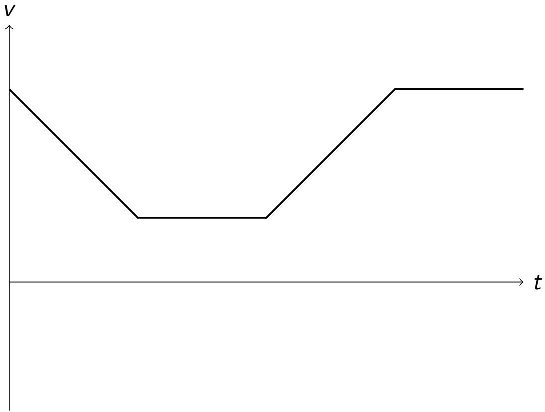
1. $a = \frac{dv}{dt} = 4 - 2t \rightarrow$ integrate: $v = \int(4 - 2t)dt = 4t - t^2 + C$. Using $v(0) = 2$ gives $C = 2$. So $v(t) = 4t - t^2 + 2$.
2. $v = \frac{ds}{dt} \rightarrow s = \int(4t - t^2 + 2)dt = 2t^2 - \frac{t^3}{3} + 2t + D$. $s(0) = 0$ gives $D = 0$. So $s(t) = 2t^2 - \frac{t^3}{3} + 2t$.
3. v–t is a downward parabola with vertex at $t = 2$ (since $dv/dt = 4 - 2t = 0$ gives $t = 2$). $v(2) = 4(2) - 4 + 2 = 6$ m/s. $v(4) = 16 - 16 + 2 = 2$ m/s. s–t is cubic.





Question 11

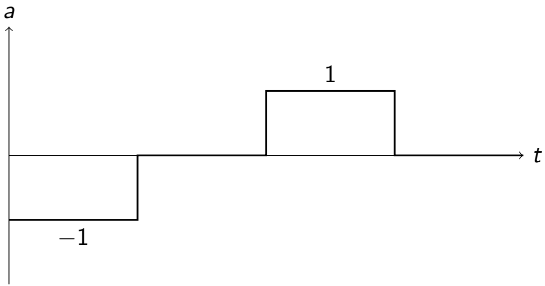
The velocity–time graph of a particle is shown. Sketch the corresponding acceleration–time graph.



Solution to Question 11

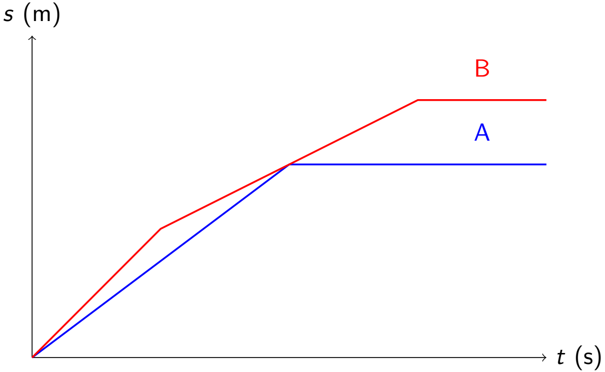
Acceleration is gradient of v–t:

- 0–2 s: gradient $= (1-3)/2 = -1$ m/s^2 $\rightarrow$ constant $a = -1$.
- 2–4 s: gradient $= 0 \rightarrow a = 0$.
- 4–6 s: gradient $= (3-1)/2 = 1$ m/s^2 $\rightarrow$ constant $a = 1$.
- 6–8 s: gradient $= 0 \rightarrow a = 0$.



Question 12

The graph shows the displacement of two runners A and B starting from the same point.



Determine the time when B overtakes A and their velocities at that instant.

Solution to Question 12

Displacement functions:

$$s_A = \begin{cases} \frac{3}{4}t, & 0 \le t \le 4 \\ 3, & t \ge 4 \end{cases}$$

$$s_B = \begin{cases} t, & 0 \le t \le 2 \\ \frac{1}{2}t + 1, & 2 \le t \le 6 \\ 4, & t \ge 6 \end{cases}$$

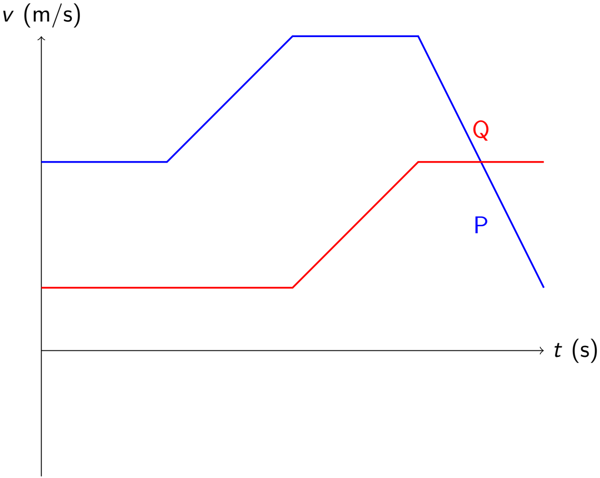
Setting $s_A = s_B$ for $t \le 2$: $\frac{3}{4}t = t \Rightarrow t = 0$. For $2 \le t \le 4$: $\frac{3}{4}t = \frac{1}{2}t + 1 \Rightarrow \frac{1}{4}t = 1 \Rightarrow t = 4$ s. At $t = 4$, $s_A = s_B = 3$ m. For $t > 4$, $s_A = 3$, $s_B > 3$ (since s_B increases to 4), so B is always ahead after $t = 4$. Thus they meet only at $t = 4$, but B was ahead before $t = 4$ and remains ahead after; therefore B never overtakes A—they are just momentarily together. Velocities at $t = 4$:

- $v_A = \frac{3}{4} = 0.75$ m/s (constant for $t < 4$).
- $v_B = \frac{1}{2}$ m/s (from $2 \le t \le 6$).

Thus at $t = 4$, $v_A = 0.75$ m/s, $v_B = 0.5$ m/s.

Question 13 (Relative motion)

Two cars P and Q move along the same straight road. Their velocity–time graphs are shown.



At $t = 0$, they are alongside. Find the time when they are again alongside.

Solution to Question 13

Compute displacement functions by integrating velocity. For P:

$$0-2: s_P = 3t$$

$$2-4: v_P = 3+(t-2) = t+1, \quad s_P = 6 + \int_2^t (t+1)dt = 0.5t^2 + t + 2$$

$$4-6: v_P = 5, \quad s_P = 14 + 5(t-4) = 5t - 6$$

$$6-8: v_P = 5 - 2(t-6) = 17 - 2t, \quad s_P = 24 + \int_6^t (17-2\tau)d\tau = 17t - t^2 - 42$$

For Q:

$$0-4: s_Q = t$$

$$4-6: v_Q = 1 + (t-4) = t-3, \quad s_Q = 4 + \int_4^t (t-3)dt = 0.5t^2 - 3t + 8$$

$$6-8: v_Q = 3, \quad s_Q = 8 + 3(t-6) = 3t - 10$$

Set $s_P = s_Q$ for $t \geq 6$ (since for earlier intervals they are not equal except at $t = 0$):

$$17t - t^2 - 42 = 3t - 10 \Rightarrow -t^2 + 14t - 32 = 0 \Rightarrow t^2 - 14t + 32 = 0$$

$$t = \frac{14 \pm \sqrt{196-128}}{2} = \frac{14 \pm \sqrt{68}}{2} = \frac{14 \pm 8.246}{2}$$

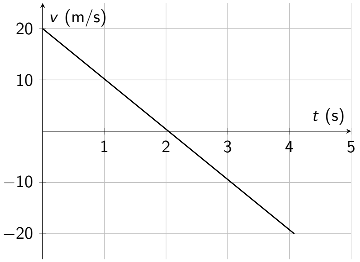
$t = 11.123$ s or $t = 2.877$ s (but $t = 2.877$ is not in $t \geq 6$). Thus they meet again at $t \approx 11.1$ s.

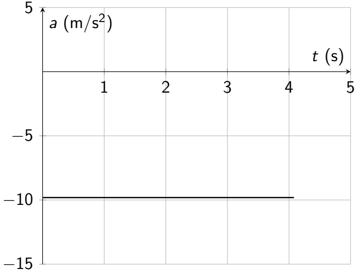
Question 14 (Projectile)

A ball is thrown vertically upward with initial speed 20 m/s. Sketch the velocity–time and acceleration–time graphs for the entire flight until it returns to the thrower's hand. Take upward as positive.

Solution to Question 14

Upward positive, $g = -9.81$ m/s^2 constant. Velocity: $v = 20 - 9.81t$. Graph is straight line from $(0, 20)$ to $(t_{top}, 0)$ where $t_{top} = 20/9.81 \approx 2.04$ s. Then continues negative until $t_{total} = 2t_{top} \approx 4.08$ s, at which $v = -20$ m/s. Acceleration: constant at -9.81 m/s^2.





Question 15 (Terminal velocity)

A skydiver jumps from a high platform. Initially she accelerates downward, but air resistance increases with speed until she reaches terminal velocity. Sketch qualitatively the v–t and a–t graphs.

Solution to Question 15

v–t: starts from rest, increases with decreasing slope (since acceleration decreases), asymptotically approaching terminal velocity v_T. a–t: starts at g, decreases to 0 as terminal velocity is approached.