H2 Physics
Circular Motion

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Brain Dump

What do you know about Circular Motion?

Brain Dump (CONT'D)

What do you know about Circular Motion?

- Angular displacement θ (radians)
- Angular velocity $\omega = \frac{d\theta}{dt}$
- Relationship $v = r\omega$
- Centripetal acceleration $a_c = \frac{v^2}{r} = r\omega^2$
- Centripetal force $F_c = \frac{mv^2}{r} = mr\omega^2$
- Sources of centripetal force: tension, friction, gravity, normal force
- Applications: horizontal circles, vertical circles, banked curves, conical pendulum
- Non-uniform circular motion (tangential acceleration)

Math Checklist

Before tackling Circular Motion, ensure you are comfortable with:

- Radians and degree-radian conversion
- Trigonometric functions (sin, cos, tan)
- Solving equations with ω, T, f
- Vector components
- Differentiation of sin and cos (for variable speed)
- Small angle approximations
- Quadratic equations
- Graphing sinusoidal functions

Building Intuition – Real-world Applications

- Car rounding a curve: friction provides centripetal force; if too fast, skidding occurs.
- Amusement park rides: loop-the-loop, rotor, gravitron – all rely on centripetal forces.
- Satellites in orbit: gravity provides centripetal force; orbital speed determines radius.
- Centrifuge: spinning rapidly to separate components based on density.
- Conical pendulum: used in governors for engines.
- Banked roads: design allows cars to turn at higher speeds without relying solely on friction.

Formalization – Angular Quantities

Angular Displacement θ

Angle swept by the radius vector, measured in radians. $1 \text{ rad} = \frac{\text{arc length}}{\text{radius}}$.

Angular Velocity ω

Rate of change of angular displacement: $\omega = \frac{d\theta}{dt}$. For uniform circular motion, $\omega = \frac{2\pi}{T} = 2\pi f$.

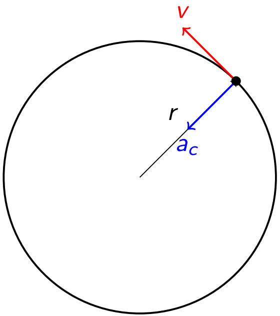
Relation between Linear and Angular Quantities

Arc length $s = r\theta$, speed $v = r\omega$, tangential acceleration $a_t = r\alpha$ (where α is angular acceleration).

Formalization – Centripetal Acceleration

For an object moving in a circle of radius r with constant speed v, the velocity vector changes direction continuously. The acceleration is directed toward the centre:

$$a_c = \frac{v^2}{r} = r\omega^2$$



Derivation: Consider velocity vectors at two nearby points; the change Δv points toward centre; $\frac{\Delta v}{v} \approx \frac{\Delta s}{r}$, so $a = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t} = \frac{v^2}{r}$.

Formalization – Centripetal Force

By Newton's second law, a net force toward the centre is required:

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

This force is not a new type of force; it is provided by existing forces such as:

- Tension (e.g., object on a string)
- Friction (e.g., car turning on flat road)
- Gravity (e.g., satellite in orbit)
- Normal force (e.g., banked curve, rotor ride)
- Combination of forces (e.g., vertical circle)

Formalization – Vertical Circular Motion

At different points, the centripetal force is provided by combinations of weight and tension/normal force.

Top of circle:

$$T + mg = \frac{mv^2}{r}$$

Minimum speed to complete circle: when $T = 0$, $v_{\text{min}} = \sqrt{gr}$.

Bottom of circle:

$$T - mg = \frac{mv^2}{r}$$

Tension is maximum here.

Energy conservation often used to relate speeds at different points.

Formalization – Banked Curves

For a frictionless banked curve with angle θ, the horizontal component of normal force provides centripetal force:

$$N \sin\theta = \frac{mv^2}{r}, \quad N \cos\theta = mg$$

Dividing: $\tan\theta = \frac{v^2}{rg}$. For a given θ, there is one design speed. At other speeds, friction is needed.

Micro-Testing – Quick Checks

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5. The period of a conical pendulum is 2.0 s. Find its angular velocity.

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5. The period of a conical pendulum is 2.0 s. Find its angular velocity. $\omega = 2\pi/T = \pi\ \mathrm{rad/s} \approx 3.14\ \mathrm{rad/s}$.

NJC 2025 H2 Physics Prelim Paper 1 Q9

A small coin of mass 10 g is placed on a horizontal rotating disc at a distance 5.0 cm from the centre. The maximum frictional force between the coin and the disc is 0.20 N. Find the maximum angular speed ω before the coin slips.

- A 10 $\mathrm{rad\,s^{-1}}$
- B 14 $\mathrm{rad\,s^{-1}}$
- C 20 $\mathrm{rad\,s^{-1}}$
- D 28 $\mathrm{rad\,s^{-1}}$

Friction provides the centripetal force:

$$f_{max} = mr\omega^2$$

$$0.20 = (0.010)(0.050)\,\omega^2$$

$$0.20 = 0.0005\,\omega^2$$

$$\omega^2 = \frac{0.20}{0.0005} = 400$$

$$\omega = 20\ \mathrm{rad\,s^{-1}}$$

Answer: **C**.

NYJC 2025 H2 Physics Prelim Paper 1 Q9

A small mass attached to a light string rotates in a vertical circle of radius r. Taking g as acceleration of free fall, what is the centripetal acceleration at the lowest point if the speed at the highest point is just enough to complete the circle?

- A g
- B $2g$
- C $4g$
- D $5g$

NYJC 2025 P1 Q9 – Solution

At highest point, minimum speed $v_{\text{top}} = \sqrt{gr}$. Using energy conservation:

$$\frac{1}{2}mv_{\text{bottom}}^2 = \frac{1}{2}mv_{\text{top}}^2 + mg(2r)$$

$$v_{\text{bottom}}^2 = gr + 4gr = 5gr$$

Centripetal acceleration at bottom:

$$a_c = \frac{v_{\text{bottom}}^2}{r} = 5g$$

Answer: **D**.

RI 2025 H2 Physics Prelim Paper 1 Q8

A ball of mass 0.10 kg is attached to a string and swung in a vertical circle of radius 0.50 m. The speed at the top is 6.0 $\mathrm{m\,s^{-1}}$. Find tension in the string at the top.

- A 0.98 N
- B 6.2 N
- C 7.2 N
- D 8.2 N

RI 2025 P1 Q8 – Solution

At the top, both weight and tension point downward:

$$T + mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} - mg = \frac{0.10 \times (6.0)^2}{0.50} - 0.10 \times 9.81 = \frac{3.6}{0.50} - 0.981 = 7.2 - 0.981 = 6$$

Answer: **B**.

HCI 2025 H2 Physics Prelim Paper 1 Q9

A small mass attached to a light string rotates in a vertical circle of radius r. What is the centripetal acceleration at the lowest point if the speed at the highest point is just enough to complete the circle? (Same as NYJC Q9)

A. g
B. $2g$
C. $4g$
D. $5g$

HCI 2025 P1 Q9 – Solution

Same as NYJC Q9 solution. Answer: **D**.

Two satellites P and Q orbit Earth. P and Q are at distances R and $3R$ from Earth's surface respectively, where R is Earth's radius. Speed of P is v. What is speed of Q?

A. $\sqrt{\frac{1}{3}}\,v$
B. $\sqrt{\frac{1}{2}}\,v$
C. $\sqrt{2}\,v$
D. $\sqrt{3}\,v$

HCI 2025 P1 Q10 – Solution

Orbital radius from centre: $r_P = R + R = 2R$, $r_Q = R + 3R = 4R$. From $\frac{GMm}{r^2} = \frac{mv^2}{r}$, we get $v = \sqrt{\frac{GM}{r}}$. So $v_P = \sqrt{\frac{GM}{2R}}$, $v_Q = \sqrt{\frac{GM}{4R}} = \frac{1}{\sqrt{2}}\sqrt{\frac{GM}{2R}} = \frac{1}{\sqrt{2}}v$. Answer: **B**.

NYJC 2025 H2 Physics Prelim Paper 2 Q3

A steel sphere mass 0.30 kg suspended from a vertical spring (equilibrium length 8.5 cm). It is then set into horizontal circular motion with radius 5.0 cm, period 0.60 s, spring length L, angle θ.

(a) Explain why spring length is greater than 8.5 cm.

(b)(i) Calculate centripetal acceleration.

(b)(ii) Show $\theta = 29^\circ$.

(b)(iii) Calculate tension.

(b)(iv) Calculate spring constant.

NYJC 2025 P2 Q3 – Solution (a)

In Fig. 3.2, vertical component of tension balances weight, and horizontal component provides centripetal force. Since the tension now has both vertical and horizontal components, its magnitude is greater than the weight alone (which was the tension in Fig. 3.1). By Hooke's law, greater tension means greater extension, so spring length increases.

NYJC 2025 P2 Q3 – Solution (b)(i)

Centripetal acceleration:

$$a_c = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = 0.050\left(\frac{2\pi}{0.60}\right)^2$$

$$\omega = \frac{2\pi}{0.60} \approx 10.472 \mathrm{~rad/s}$$

$$a_c = 0.050 \times (10.472)^2 = 0.050 \times 109.66 \approx 5.48 \mathrm{~m/s^2} \approx 5.5 \mathrm{~m/s^2}$$

NYJC 2025 P2 Q3 – Solution (b)(ii)

Forces: $T \sin\theta = ma_c$, $T\cos\theta = mg$. Dividing:

$$\tan\theta = \frac{a_c}{g} = \frac{5.483}{9.81} \approx 0.559$$

$$\theta = \tan^{-1}(0.559) \approx 29.2^\circ \approx 29^\circ$$

From $T\cos\theta = mg$:

$$T = \frac{mg}{\cos\theta} = \frac{0.30 \times 9.81}{\cos 29.2^\circ} \approx \frac{2.943}{0.873} \approx 3.37\ \mathrm{N} \approx 3.4\ \mathrm{N}$$

NYJC 2025 P2 Q3 – Solution (b)(iv)

Spring constant k from $k = \frac{\Delta F}{\Delta x}$. First find L:
$\sin\theta = \frac{r}{L} \Rightarrow L = \frac{r}{\sin\theta} = \frac{0.050}{\sin 29.2^\circ} \approx \frac{0.050}{0.488} \approx 0.1025\ \mathrm{m}$. Original length when vertical (equilibrium) $L_0 = 0.085\ \mathrm{m}$. Extension in circular motion:
$x = L - L_0 = 0.1025 - 0.085 = 0.0175\ \mathrm{m}$. Force in spring = tension $T = 3.37\ \mathrm{N}$.
Original tension when vertical $= mg = 2.943\ \mathrm{N}$. Increase in force
$\Delta F = 3.37 - 2.943 = 0.427\ \mathrm{N}$. Thus $k = \frac{\Delta F}{x} = \frac{0.427}{0.0175} \approx 24.4\ \mathrm{N/m}$.

RI 2025 H2 Physics Prelim Paper 2 Q8(d)(i)

A skier of mass 75 kg carves a turn on hard snow with edge angle $\beta = 61°$, speed 5.0 $\mathrm{m\,s^{-1}}$, turn radius $r = 9.2$ m (from Fig. 8.7). Find centripetal force.

RI 2025 P2 Q8(d)(i) – Solution

Centripetal force

$F_c = \frac{mv^2}{r} = \frac{75\times(5.0)^2}{9.2} = \frac{75\times25}{9.2} = \frac{1875}{9.2} \approx 203.8\ \mathrm{N} \approx 200\ \mathrm{N}.$

HCI 2025 H2 Physics Prelim Paper 3 Q5(a)

[Prereq: E&M]

An electron moves at 20° to a magnetic field $0.088\ \mathrm{T}$ with force $4.3 \times 10^{-14}\ \mathrm{N}$. Find its speed. Then find the pitch of the helical path.

HCI 2025 P3 Q5(a) – Solution (speed)

Magnetic force $F = Bqv\sin\theta$. $q = 1.60 \times 10^{-19}$ C.

$$v = \frac{F}{Bq\sin\theta} = \frac{4.3 \times 10^{-14}}{0.088 \times 1.60 \times 10^{-19} \times \sin 20^\circ}$$

$\sin 20^\circ \approx 0.342$, denominator
$= 0.088 \times 1.60 \times 10^{-19} \times 0.342 = 0.088 \times 0.5472 \times 10^{-19} = 4.815 \times 10^{-21}$.
$v = 4.3 \times 10^{-14}/4.815 \times 10^{-21} \approx 8.93 \times 10^{6}\ \mathrm{m\,s^{-1}}$.

HCI 2025 P3 Q5(a) – Solution (pitch)

Period of circular motion $T = \frac{2\pi m}{Bq}$.

$$T = \frac{2\pi(9.11 \times 10^{-31})}{0.088 \times 1.60 \times 10^{-19}} \approx \frac{5.72 \times 10^{-30}}{1.408 \times 10^{-20}} \approx 4.06 \times 10^{-10} \text{ s}$$

Velocity parallel to field: $v_{\parallel} = v \cos 20^\circ = 8.93 \times 10^6 \times 0.9397 \approx 8.39 \times 10^6 \text{ m/s}$. Pitch $p = v_{\parallel} T = 8.39 \times 10^6 \times 4.06 \times 10^{-10} \approx 3.41 \times 10^{-3} \text{ m}$.

HCI 2025 H2 Physics Prelim Paper 3 Q5(b)

[Prereq: E&M]
A coil in a motor has area $6.1 \times 10^{-3}\ \mathrm{m}^2$, 1200 turns, current 96 A, maximum torque 395 Nm. Find magnetic flux density B.

HCI 2025 P3 Q5(b) – Solution

Maximum torque $\tau = NBIA$, where A is area.

$$B = \frac{\tau}{NIA} = \frac{395}{1200 \times 96 \times 6.1 \times 10^{-3}}$$

Denominator: $1200 \times 96 = 115200$, times $6.1 \times 10^{-3} = 702.72$.
$B = 395/702.72 \approx 0.562\ \mathrm{T}$.

Variation 1 – Car on Flat Curve Difficulty: 3/10

A car of mass 1200 kg rounds a flat curve of radius 50 m at 15 $\mathrm{m/s}$. What minimum coefficient of friction is needed to prevent skidding?

Variation 1 – Solution

Friction provides centripetal force: $f = \mu mg = \frac{mv^2}{r}$.

$$\mu = \frac{v^2}{rg} = \frac{15^2}{50 \times 9.81} = \frac{225}{490.5} \approx 0.459$$

A mass 0.2 kg on a string of length 1.0 m moves in a horizontal circle with the string making 25° to the vertical. Find the tension and the period.

Variation 2 – Solution

Vertical: $T\cos 25^\circ = mg \Rightarrow T = \frac{0.2\times 9.81}{0.9063} \approx 2.165\ \mathrm{N}$. Radius $r = L\sin 25^\circ = 1.0 \times 0.4226 = 0.4226\ \mathrm{m}$. Horizontal: $T\sin 25^\circ = m\omega^2 r \Rightarrow 2.165 \times 0.4226 = 0.2\,\omega^2 \times 0.4226$. Cancel 0.4226: $2.165 = 0.2\,\omega^2 \Rightarrow \omega^2 = 10.825$, $\omega \approx 3.29\ \mathrm{rad/s}$. Period $T = \frac{2\pi}{\omega} \approx 1.91\ \mathrm{s}$.

Variation 3 – Rotating Platform Difficulty: 3/10

A coin is placed 8.0 cm from the centre of a rotating turntable. The coefficient of static friction is 0.35. Find the maximum angular speed before the coin slips.

Variation 3 – Solution

$f_{\text{max}} = \mu mg = mr\omega^2 \Rightarrow \omega^2 = \frac{\mu g}{r} = \frac{0.35 \times 9.81}{0.08} = \frac{3.4335}{0.08} = 42.92.$
$\omega \approx 6.55 \text{ rad/s}.$

Variation 4 – Bucket of Water Difficulty: 4/10

A bucket of water is swung in a vertical circle of radius $1.2\ \mathrm{m}$. What minimum speed at the top is needed so the water doesn't fall out?

Variation 4 – Solution

At top, water just loses contact when $N = 0$, so
$mg = \frac{mv^2}{r} \Rightarrow v = \sqrt{gr} = \sqrt{9.81 \times 1.2} \approx \sqrt{11.772} \approx 3.43\ \mathrm{m/s}.$

Variation 5 – Roller Coaster Loop Difficulty: 5/10

A roller coaster car of mass 500 kg (including passengers) goes through a vertical loop of radius 10 m. If the speed at the bottom is 20 $\mathrm{m/s}$, find the normal force at the bottom and at the top (assuming no loss of energy).

Variation 5 – Solution

At bottom: $N_{\text{bottom}} - mg = \frac{mv_b^2}{r} \Rightarrow N_{\text{bottom}} = mg + \frac{mv_b^2}{r} = 500 \times 9.81 + \frac{500 \times 400}{10} = 4905 + 20000 = 24905\ \mathrm{N}$. Find speed at top using energy:
$\frac{1}{2}mv_t^2 = \frac{1}{2}mv_b^2 - mg(2r) \Rightarrow v_t^2 = 400 - 4 \times 9.81 \times 10 = 400 - 392.4 = 7.6$, so $v_t \approx 2.76\ \mathrm{m/s}$. At top:
$N_{\text{top}} + mg = \frac{mv_t^2}{r} \Rightarrow N_{\text{top}} = \frac{500 \times 7.6}{10} - 4905 = 380 - 4905 = -4525\ \mathrm{N}$ (negative means force is downward, i.e., track pushes down? Actually normal force can't be negative; if calculation gives negative, it means the car would lose contact unless it's held by something. Here v_t is less than $\sqrt{gr} = 9.9$, so it would fall. So this speed is too low.)

Variation 6 – Minimum Speed at Top Difficulty: 4/10

For the roller coaster in Variation 5, what minimum speed at the top is needed to just maintain contact?

Variation 6 – Solution

$$v_{\text{min}} = \sqrt{gr} = \sqrt{9.81 \times 10} \approx \sqrt{98.1} \approx 9.90 \ \mathrm{m/s}.$$

Variation 7 – Banked Curve (no friction) Difficulty: 4/10

A highway curve is banked at 15° with radius 80 m. Find the design speed (no friction required).

Variation 7 – Solution

$\tan\theta = \frac{v^2}{rg} \Rightarrow v^2 = rg\tan\theta = 80 \times 9.81 \times \tan 15^\circ$. $\tan 15^\circ \approx 0.2679$, so $v^2 = 80 \times 9.81 \times 0.2679 \approx 210.2$, $v \approx 14.5\ \mathrm{m/s}$.

Variation 8 – Banked Curve with Friction Difficulty: 6/10

A curve of radius 100 m is banked at 10°. If the coefficient of static friction is 0.30, find the maximum safe speed.

Variation 8 – Solution

Maximum speed when friction acts down the slope. Equations: Horizontal: $N \sin\theta + f \cos\theta = \frac{mv^2}{r}$. Vertical: $N \cos\theta - f \sin\theta = mg$, with $f = \mu N$. Substitute f and solve for v: From vertical: $N(\cos\theta - \mu \sin\theta) = mg \Rightarrow N = \frac{mg}{\cos\theta - \mu \sin\theta}$. Horizontal: $N(\sin\theta + \mu \cos\theta) = \frac{mv^2}{r}$. Divide horizontal by vertical: $\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$.
$v^2 = rg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$. $\sin 10^\circ \approx 0.1736$, $\cos 10^\circ \approx 0.9848$. Numerator $= 0.1736 + 0.30 \times 0.9848 = 0.1736 + 0.2954 = 0.4690$. Denominator $= 0.9848 - 0.30 \times 0.1736 = 0.9848 - 0.05208 = 0.9327$.
$v^2 = 100 \times 9.81 \times \frac{0.4690}{0.9327} = 981 \times 0.5028 \approx 493.2$, $v \approx 22.2\ \mathrm{m/s}$.

Challenge 1 – Loop with Friction Difficulty: 8/10

A small block starts from rest at height h above the bottom of a circular loop of radius R. The track has coefficient of kinetic friction μ on the horizontal portion only (the loop itself is frictionless). Find h such that the block just makes it around the loop.

Challenge 1 – Solution

First find speed needed at top of loop: $v_{\text{top}} = \sqrt{gR}$. Energy from start to top: $mgh = mg(2R) + \frac{1}{2}mv_{\text{top}}^2 +$ work against friction. Work against friction on horizontal portion depends on how many times it traverses. Usually the block enters the loop directly, so no horizontal section before loop. If there is a horizontal section before the loop, then work done $=$ μmgd where d is length. Then $h = 2R + \frac{1}{2}R + \mu d = \frac{5}{2}R + \mu d$.

Challenge 2 – Bead on Rotating Hoop Difficulty: 9/10

A small bead slides on a circular hoop of radius R rotating about a vertical diameter with constant angular velocity ω. Find the angle θ (from vertical) at which the bead remains stationary relative to the hoop. Discuss the stability.

Challenge 2 – Solution

In rotating frame, effective potential:
$U_{\text{eff}} = mgR(1 - \cos\theta) - \frac{1}{2}m\omega^2 R^2 \sin^2\theta$. Equilibrium when $\frac{dU_{\text{eff}}}{d\theta} = 0$: $mgR\sin\theta - m\omega^2 R^2 \sin\theta\cos\theta = 0$. Either $\sin\theta = 0$ (bottom, $\theta = 0$), or $\cos\theta = \frac{g}{\omega^2 R}$. For $\omega^2 R > g$, there are two symmetric positions. Stability determined by second derivative.

Challenge 3 – Car on Banked Curve with Varying Radius Difficulty: 8/10

A car travels on a spiral curve where the radius r decreases linearly with distance travelled. Derive an expression for the required banking angle as a function of r if no friction is needed at constant speed v.

Challenge 3 – Solution

For no friction, $\tan\theta = \frac{v^2}{rg}$. If v is constant, then $\theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$. As r decreases, θ increases. The road must be banked more sharply for tighter turns.

Challenge 4 – Mass on String in Horizontal Circle with Variable Length Difficulty: 8/10

A mass is attached to a string that is being pulled down through a hole in the table at constant speed u, so the radius of the circle decreases. If the initial angular velocity is ω_0 at radius r_0, find ω as a function of r. Assume no friction.

Challenge 4 – Solution

No torque about the centre (tension is radial), so angular momentum $L = mr^2\omega$ is conserved. Thus $r^2\omega = \text{constant} = r_0^2\omega_0$, so $\omega = \omega_0 \left(\frac{r_0}{r}\right)^2$.

Challenge 5 – Orbit Transfer (Hohmann) Difficulty: 9/10

A satellite in circular orbit of radius r_1 around Earth transfers to a higher circular orbit of radius r_2 via an elliptical transfer orbit. Find the required change in speed at perigee and apogee.

Challenge 5 – Solution

In circular orbit: $v_1 = \sqrt{\frac{GM}{r_1}}$, $v_2 = \sqrt{\frac{GM}{r_2}}$. Elliptical transfer orbit has semi-major axis $a = \frac{r_1+r_2}{2}$. At perigee (closest, distance r_1), speed $v_p = \sqrt{GM\left(\frac{2}{r_1} - \frac{1}{a}\right)} = \sqrt{2GM\left(\frac{1}{r_1} - \frac{1}{r_1+r_2}\right)}$. $\Delta v_1 = v_p - v_1$. At apogee (distance r_2), speed $v_a = \sqrt{GM\left(\frac{2}{r_2} - \frac{1}{a}\right)}$, and $\Delta v_2 = v_2 - v_a$.

Variation 9 – Whirling a Bucket Difficulty: 3/10

A bucket of water is whirled in a vertical circle of radius $1.0\ \mathrm{m}$. What is the minimum speed at the top so the water doesn't spill?

Variation 9 – Solution

$$v_{\text{min}} = \sqrt{gr} = \sqrt{9.81 \times 1.0} \approx 3.13 \text{ m/s}.$$

Variation 10 – Rotor Ride Difficulty: 5/10

In a rotor ride, a person stands against the wall of a spinning cylinder of radius 3.0 m. The coefficient of static friction between person and wall is 0.40. Find the minimum angular speed so the person doesn't slide down when the floor drops.

Variation 10 – Solution

Forces: friction $f = \mu N$ upward balances weight mg. Normal force N provides centripetal force: $N = mr\omega^2$. Thus $f = \mu mr\omega^2 \geq mg \Rightarrow \omega^2 \geq \frac{g}{\mu r} = \frac{9.81}{0.40 \times 3.0} = \frac{9.81}{1.2} = 8.175$, $\omega \geq 2.86\ \mathrm{rad/s}$.

Variation 11 – String Breaking Difficulty: 4/10

A string can withstand a maximum tension of 50 N. A 0.5 kg mass is whirled in a horizontal circle of radius 0.8 m. Find the maximum speed possible.

Variation 11 – Solution

$$T_{\max} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{T_{\max} r}{m} = \frac{50\times 0.8}{0.5} = \frac{40}{0.5} = 80,\ v \approx 8.94\ \mathrm{m/s}.$$

Variation 12 – Satellite Period Difficulty: 3/10

A satellite orbits Earth at a height of 200 km above the surface. Earth's radius $R = 6400\ \mathrm{km}$, mass $M = 6.0 \times 10^{24}\ \mathrm{kg}$, $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$. Find the orbital period.

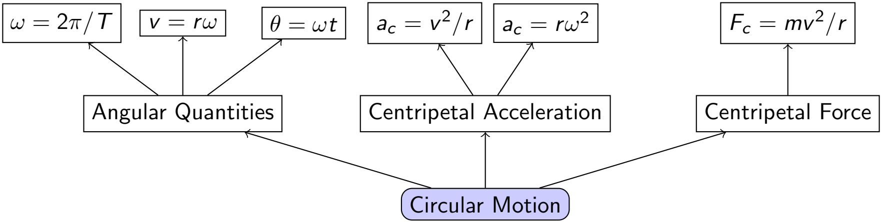
Variation 12 – Solution

Orbital radius $r = 6400 + 200 = 6600\ \mathrm{km} = 6.6 \times 10^6\ \mathrm{m}$. $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6.6\times10^6}} = \sqrt{\frac{4.002\times10^{14}}{6.6\times10^6}} = \sqrt{6.064 \times 10^7} \approx 7788\ \mathrm{m/s}$. Period $T = \frac{2\pi r}{v} = \frac{2\pi\times6.6\times10^6}{7788} \approx \frac{4.146\times10^7}{7788} \approx 5324\ \mathrm{s} \approx 88.7\ \mathrm{min}$.

End-of-Session Concept Recap

- Circular motion requires a net centripetal force directed toward the centre.
- Angular velocity $\omega = 2\pi/T = 2\pi f$.
- Relationships: $v = r\omega$, $a_c = v^2/r = r\omega^2$, $F_c = mv^2/r = mr\omega^2$.
- Centripetal force is provided by tension, friction, gravity, normal force, or combinations.
- In vertical circles, speed varies; use energy conservation to relate speeds at different points.
- Banked curves allow turning without friction at design speed $\tan\theta = v^2/(rg)$.
- Satellites: $GMm/r^2 = mv^2/r$ gives orbital speed $v = \sqrt{GM/r}$.

Mind Map



Link to dynamics: centripetal force is the net force causing circular motion.
Link to gravitation: $F = GMm/r^2$ provides centripetal force for orbits.